1.0 Objective:

After reading this unit you will be able to understand:
* Defining vector.
*Vector representation, addition, subtraction
*Orthogonal representation
*Multiplication of vectors
*Scalar product, vector product
*Scalar triple product and vector triple product

1.1 Introduction:

On the basis of direction, the physical quantities may be divided into two main classes.

1.1.1 Scalar quantities: The physical quantities which do not require direction for their representation. These quantities require only magnitude and unit and are added according to the usual rules of algebra. Examples of these quantities are: mass, length, area, volume, distance, time speed, density, electric current, temperature, work etc.

1.1.2 Vector quantities: The physical quantities which require both magnitude and direction and which can be added according to the vector laws of addition are called vector quantities or vector. These quantities require magnitude, unit and direction. Examples are weight, displacement, velocity, acceleration, magnetic field, current density, electric field, momentum angular velocity, force etc.

1.2 Vector representation:

Any vector quantity say A, is represented by putting a small arrow above the physical quantity like $\vec{A}$. In case of print text a vector quantity is represented by bold type letter like **A**. The vector can be represented by both capital and small letters. The magnitude of a vector quantity A is denoted by $|\vec{A}|$ or *mod A* or some time light forced italic letter *A*. We should understand following types of vectors and their representations.

1.2.1 Unit vector

A unit vector of any vector quantity is that vector which has unit magnitude. Suppose $\vec{A}\ is\ a\ vector$ then unit vector is defined as

$$\hat{A} = \frac{\vec{A}}{|A|}$$

The unit vector is denoted by $\hat{A}$ and read as 'A *unit vector or A hat'*. It is clear that the magnitude of unit vector is always 1. A unit vector merely indicates direction only. In Cartesian coordinate system, the unit vector along x, y and x axis are represented by $\hat{\imath}$, $\hat{\jmath}$ and $\hat{k}$ respectively as shown in figure 1.1.

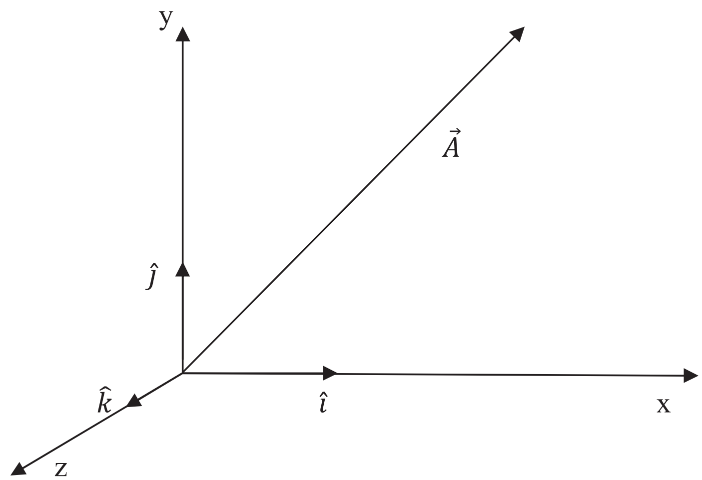

Figure 1.1

Any vector in Cartesian coordinate system can be represented as

$$A = \hat{\imath}\, A_x + \hat{\jmath}\, A_y + \hat{k}\, A_z$$

Where $\hat{\imath}$, $\hat{\jmath}$ and $\hat{k}$ are unit vector along x, y z axis and, A_x , A_y , A_z are the magnitudes projections or components of $\vec{A}$ along x, y, z axis respectively.

The unit vector in Cartesian coordinate system can be given as:

$$\hat{A} = \frac{\hat{\imath}\, A_x + \hat{\jmath}\, A_y + \hat{k}\, A_z}{\sqrt{{A_x}^2 + {B_x}^2 + {C_x}^2}}$$

1.2.2. Zero vector or Null vector:

A vector with zero magnitude is called zero vector or null vector. The condition for null vector is $|\vec{A}| = 0$

1.2.3 Equal vectors:

If two vectors have same magnitude and same direction, the vectors are called equal vector.

1.2.4 Like vectors:

If two or more vectors have same direction, but may have different magnitude, then the vectors are called like vectors.

1.2.5 Negative vector:

A vector is called negative vector with reference to another one, if both have same magnitude but opposite directions.

1.2.6 Collinear vectors:

All the vectors parallel to each other are called collinear vectors. Basically collinear means the line of action is along the same line.

1.2.6 Coplanar vector:

All the vectors whose line of action lies on a same plane are called coplanar vectors. Basically coplanar means lies on the same plane.

1.2.7. Graphical representation of vectors:

Graphically a vector quantity is represented by an arrow shaped straight line, with suitable length which represents magnitude, and the direction of arrow represents direction of vector quantity. For example, if a force $\vec{A}$ is directed towards east and another force $\vec{B}$ is directed toward north-west then these forces can be represented as shown in figure 1.2.

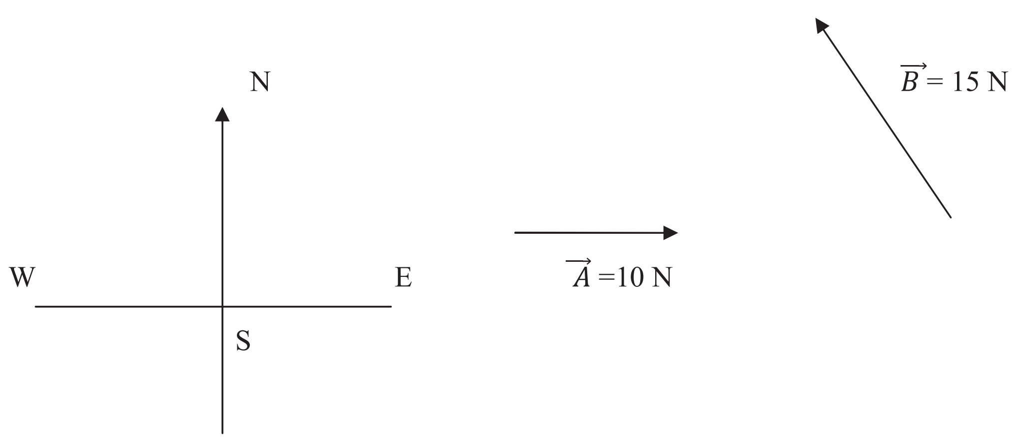

Figure 1.2

1.2.8 Addition and subtraction of vectors:

The addition of two vectors can be performed by following two laws.

(A) The parallelogram law:

According to this law, if two vectors $\vec{A}$ and $\vec{B}$ are represented by two adjacent sides of a parallelogram as show in figure 1.3, then the sum of these two vectors or resultant $\vec{R}$ is represented by the diagonal of Parallelogram.

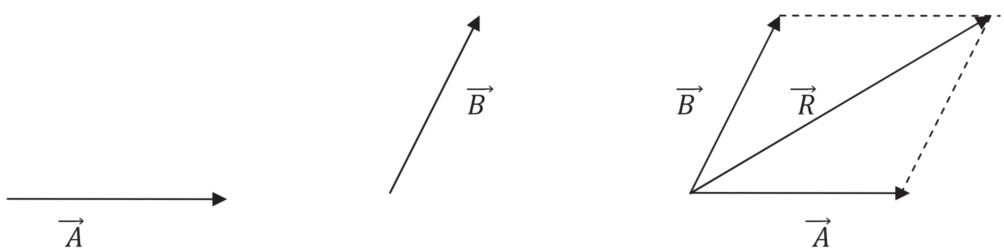

Figure 1.3

If vector $\vec{A}$ and $\vec{B}$ are represented by the sides of a parallelogram as shown in figure 1.4 and the angle between $\vec{A}$ and $\vec{B}$ is θ, and resultant $\vec{R}$ makes angle α with vector $\vec{A}$ then magnitude of $\vec{R}$ is

$$|R| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

The angle α is given as

$$\alpha = tan^{-1}\frac{B\sin\theta}{A + Bcos\,\theta}$$

You should notice that all three vectors $\vec{A}$, $\vec{B}$ and $\vec{R}$ are concurrent *i.e.* vectors acting on the same point O.

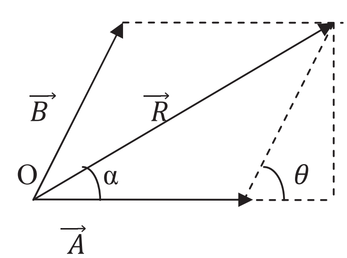

Figure 1.4

(B) Triangle law:

According to this law if a vector is placed at the head of another vector, and these two vectors represent two sides of a triangle then the third side or a vector drawn for the tail end of first to the head end of second represents the resultant of these two vectors. If vectors $\vec{A}$ and $\vec{B}$ are two vector as shown in figure 1.5, then resultant $\vec{R}$ can be obtained by applying triangle law.

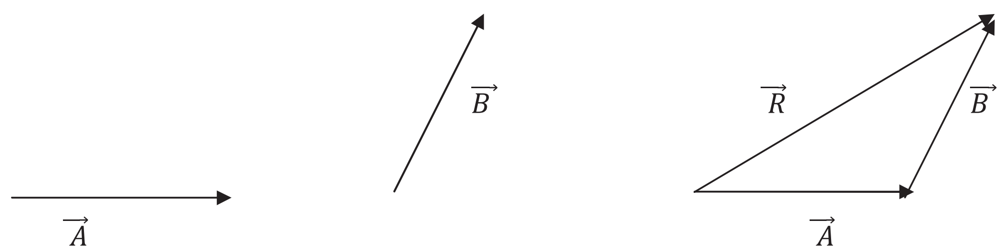

Figure 1.5

(c) Polygon law of vector addition:

This law is used for the addition of more than two vectors. According to this law if we have a large number of vectors, place the tail end of each successive vector at the head end of previous one. The resultant of all vectors can be obtained by drawing a vector from the tail end of first to the head end of the last vector. Figure 1.6 shows the polynomial law of vector addition different vectors $\vec{A}$,$\vec{B}$, $\vec{C}$, $\vec{D}$, $\vec{E}$ etc. and $\vec{R}$ is resultant vector.

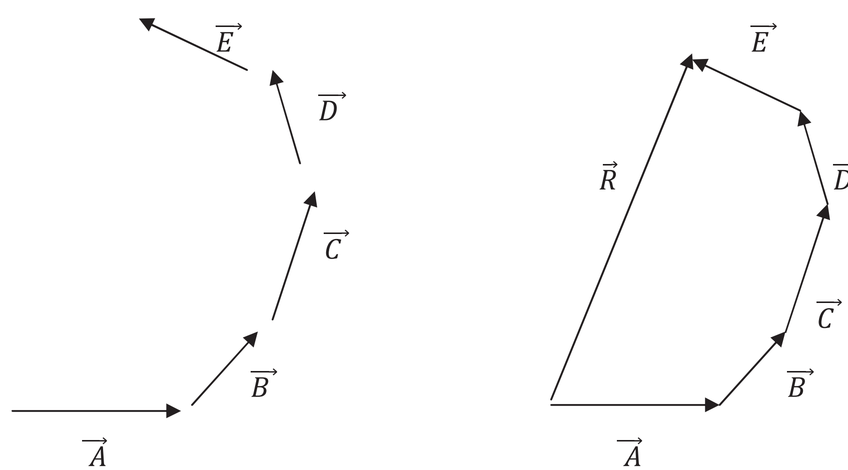

Figure 1.6

1.2.9 Resolution of vector:

A vector can be resolved into two or more vectors and these vectors can be added in accordance with the polygon law of vector addition, and finally original vector can be obtained. If a vector is resolved into three components which are mutually perpendicular to each other then these are called rectangular components or mutual perpendicular components of a vector. These components are along the three coordinate axes x, y and z respectively as show in figure 1.7.

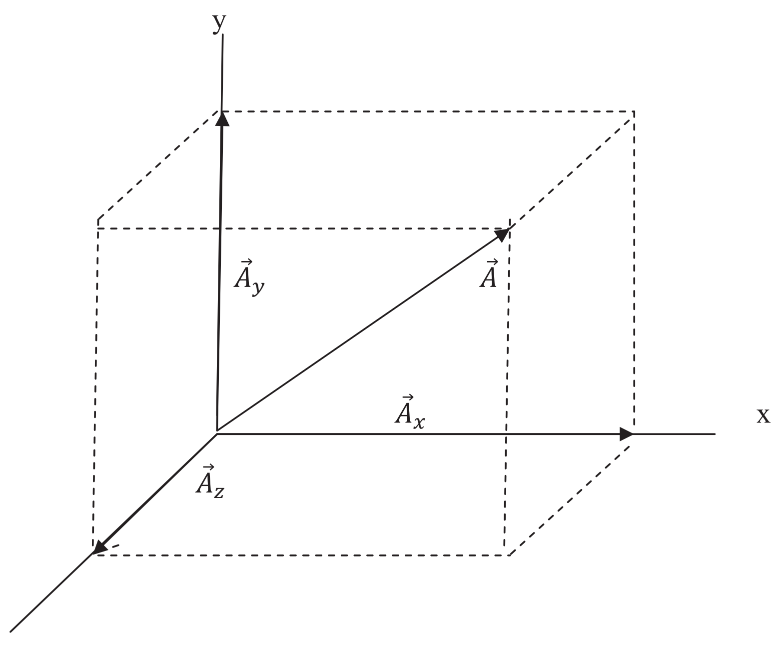

Figure 1.7

If the unit vectors along x, y and x axis are represented by $\hat{\imath}$, $\hat{\jmath}$ and $\hat{k}$ respectively then any vector $\vec{A}$ can be give as

$$\vec{A} = \hat{\imath}\, A_x + \hat{\jmath}\, A_y + \hat{k}\, A_z$$

$\vec{A}$ constitutes the diagonal of a parallelepiped, and A_x , A_y and A_z are the edges along x, y and z axes respectively. $\vec{A}$ is polynomial addition of vectors A_x , A_y and A_z . The rectangular components A_x , A_y and A_z can be considered as orthogonal projections of vector $\vec{A}$ on x, y and z axis respectively. Mathematically, the magnitude of vector $\vec{A}$ can be given as:

$$\boldsymbol{A} = |\vec{A}| = \sqrt{A_x{}^2 + A_y{}^2 + A_z{}^2}$$

1.2.10 Direction cosines:

The cosine of angles which the vector $\vec{A}$ makes with three mutual perpendicular axes x, y and z are called direction cosine and generally represented by l, m, n respectively. In figure 1.8 vector $\vec{A}$ makes angle α, β *and* γ with axis x, y and z respectively. Then

$$cos\ \alpha = \frac{A_x}{A} = \frac{A_x}{\sqrt{A_x{}^2 + A_y{}^2 + A_z{}^2}};\ cos\ \beta = \frac{A_y}{A} = \frac{A_y}{\sqrt{A_x{}^2 + A_y{}^2 + A_z{}^2}};$$

$$cos\ \gamma = \frac{A_z}{A} = \frac{A_z}{\sqrt{A_x{}^2 + A_y{}^2 + A_z{}^2}}$$

Where A_x, A_y and A_z are the projection or intercepts of vector $\vec{A}$ along x, y and z axes respectively. The $cos\ \alpha$, $cos\ \beta$ and $cos\ \gamma$ are called direction cosines.

$$l = cos\ \alpha;\quad m = cos\ \beta\ ;\ n = cos\ \gamma$$

Mathematically

$$cos^2\alpha + cos^2\beta + cos^2\gamma = 1 \text{ or } \quad l^2 + m^2 + n^2 = 1$$

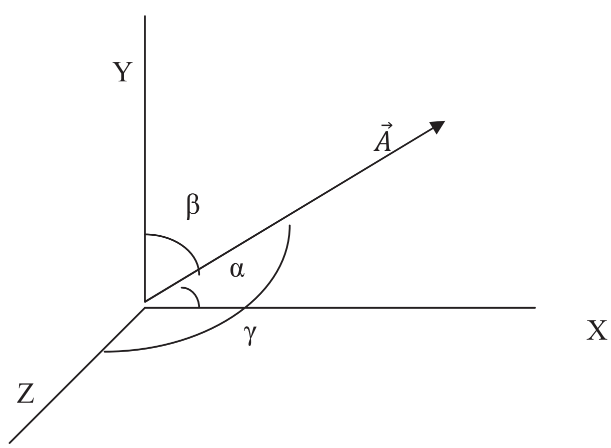

Figure 1.8

1.2.11 Position vector:

In Cartesian co-ordinate system the position of any point P(x, y, z) can be represented by a vector **r**, with respect to origin O then the vector **r** is called position vector of point P. Position vector is often denoted by $\bar{r}$. Figure1.9 shows the position vector of a point P in Cartesian coordinate system. If we have two vectors $\vec{P}$ and $\vec{Q}$ with position vectors $\boldsymbol{r_1}$ and $\boldsymbol{r_2}$ respectively then

$$\boldsymbol{r_1} = \hat{\imath}\, x_1 + \hat{\jmath}\, y_1 + \hat{k}\, z_1$$

$$\boldsymbol{r_2} = \hat{\imath}\, x_2 + \hat{\jmath}\, y_2 + \hat{k}\, z_2$$

Where (x_1, y_1, z_1) and (x_2, y_2, z_2) are the coordinates of point P and Q respectively.

Now the vector PQ can be given as

PQ = OQ – OP (∴ OP + PQ = OQ)

$$\bar{r} = \bar{r_2} - \bar{r_1}$$

Therefore, vector PQ = position vector of Q − position vector of P

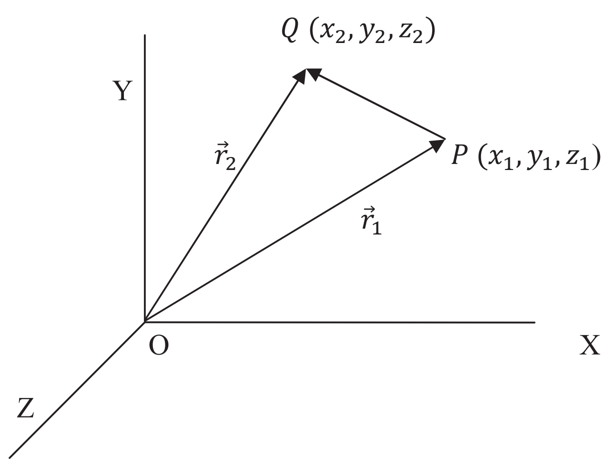

Figure1.9

1.3 Multiplication of vectors:

1.3.1 Multiplication and division of a vector by scalar:

If a vector **P** is multiplied by a scalar quantity m then its magnitude becomes m times. For example if m is a scalar and $\vec{A}$ is a vector then its magnitude becomes m times. Similarly, in case of division of a vector A by a non zero scalar quantity n, its magnitude becomes 1/n times.

1.3.2 Product of two vectors:

There are two distinct ways in which we can define the product of two vectors.

1.3.2.1 Scalar product or dot product:

Scalar product of two vectors **P** and **Q** is defined as the product of magnitude of two vectors P and Q and cosine of the angle between the directions of these vectors.

If θ is the angle between two vectors $\vec{P}$ and $\vec{Q}$, then dot product (*read as* $\vec{P}\ dot\ \vec{Q}$) of two vectors is given by-

$$\overrightarrow{P}.\vec{Q} = PQ\cos\theta = P\,(Q\cos\theta)$$

$$= P\,(projection\ of\ vector\ Q\ on\ P) = P.MN$$

The figure 1.10 shows the dot product. The resultant of dot product or scalar product of two vectors is always a scalar quantity. In physics the dot product is frequently used, the simplest example is work which is dot product of force and displacement vectors.

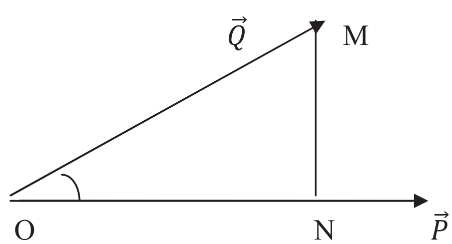

Figure1.10

Important properties of dot product

(i) Condition for two collinear vectors:

If two vectors are parallel or angle between two vectors is 0 or π, then vectors are called collinear. In this case

$$\overrightarrow{P}.\vec{Q} = PQcos\ 0^o = PQ$$

Then the product of two vectors is same as the product of their magnitudes.

(ii) Condition for two vector to be perpendicular to each other:
If two vectors are perpendicular to each other then the angle between these two vectors is 90^0, then

$$\overrightarrow{P}.\vec{Q} = PQcos\ 90^o = 0$$

Hence two vectors are perpendicular to each other if and only if their dot product is zero.

In case of unit vectors $\hat{\imath}$, $\hat{\jmath}$ and $\hat{k}$ we know that these vectors are perpendicular to each other then

$\hat{\imath}\,.\,\hat{\jmath} = \hat{\imath}.\hat{k} = \hat{k}.\hat{\imath} = 0$

similarly

$\hat{\imath}\,.\,\hat{\imath} = \hat{\jmath}.\hat{\jmath} = \hat{k}.\hat{k} = 1$

(iii) Commutative law holds:
In case of vector dot product the commutative law holds. Then

$$\overrightarrow{P}.\vec{Q} = \vec{Q}.\vec{P}$$

(iv) Distributive property of scalar product:
If P, Q and R are three vectors then according to distributive law

$\vec{P}.(\vec{Q}+\vec{R}) = \vec{P}.\vec{Q} + \vec{P}.\vec{R}$

Example 1.1 Show that vector $\vec{A} = 3i + 6j - 2k$ and $\vec{B} = 4i - \hat{\jmath} + 3k$ are mutually perpendicular.

Solution: If the angle between $\vec{A}$ and $\vec{B}$ is θ then

$\vec{A}\,.\,\vec{B} = A\,B\cos\theta$

$\cos\theta = \frac{\vec{A}.\vec{B}}{AB} = \frac{(3i+6j-2k).(4i-j+3k)}{\sqrt{(A_x^2+A_y^2+A_z^2)}\sqrt{(B_x^2+B_y^2+B_z^2}} = 0$

$\cos\theta = 0, \theta = 90^0$

Therefore the vectors are mutually perpendicular.

Example 1.2 A particle moves from a point (3,-4,-2) meter to another point (5,-6, 2) meter under the influence of a force $\vec{F} = (-3\hat{\imath} + 4\hat{J} + 4\hat{k})$ N. Calculate the work done by the force.

Solution: Suppose the particle moves from point A to B. Then displacement of particle is given by

$$\vec{r} = position\ vetor\ of\ B - position\ vetor\ A$$
$$\vec{r} = [(5-3)i + (-6+4)j + (2+2)k] \text{ meter}$$
$$\vec{r} = (2i - 2j + 4k) \text{ meter}$$

Work done = $\vec{F}.\vec{r}$ = $[(-3\hat{\imath} + 4\hat{\jmath} + 4\hat{k}).(2i - 2j + 4k)$ N-meter =2 joule.

1.3.2.2 Vector product or Cross Product

The vector product or cross product of two vectors is a vector quantity and defined as a vector whose magnitude is equal to the product of magnitudes of two vectors and sine of angle between them.

If $\vec{A}$ and $\vec{B}$ are two vectors then cross product of these two vectors is denoted by $\vec{A} \times \vec{B}$ (*read as* $\vec{A}\ cross\ \vec{B}$) and given as

$$\vec{A} \times \vec{B} = ABsin\emptyset\ \hat{n} = \vec{C}$$

Where $\emptyset$ is the angle between vectors $\vec{A}$ and $\vec{B}$, and $\hat{n}$ is the unit vector perpendicular to both $\vec{A}$ and $\vec{B}\left(i.e. normal\ to\ the\ plane\ containg\ \vec{A}\ and\ \vec{B}\right)$.

Suppose $\vec{A}$ is along x axis and $\vec{B}$ is along y axis then vector product can be considered as an area of parallelogram OPQR as shown is figure 1.11 in XY plane whose sides are $\vec{A}$ and $\vec{B}$ and direction is perpendicular to plane OPQR i.e. along z axis. The cross product $\vec{A}$ and $\vec{B}$ is positive if direction of $\emptyset$ ($\vec{A}\ to\ \vec{B}$) is positive or rotation is anticlockwise as show in figure 1.11, and negative if the rotation of $\emptyset$ ($\vec{A}\ to\ \vec{\boldsymbol{B}}$) is clockwise (figure 1.12).

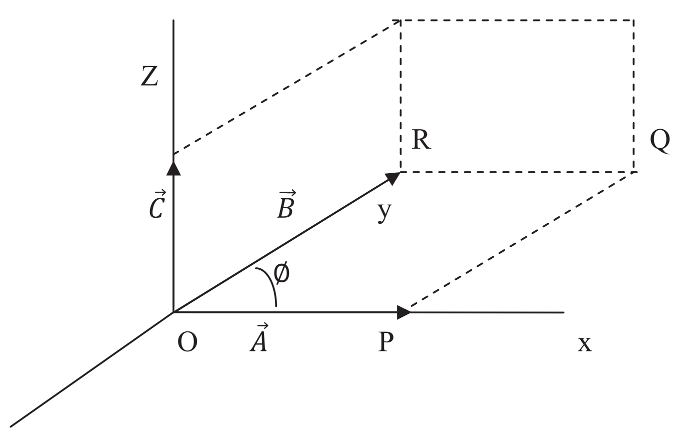

Figure1.11

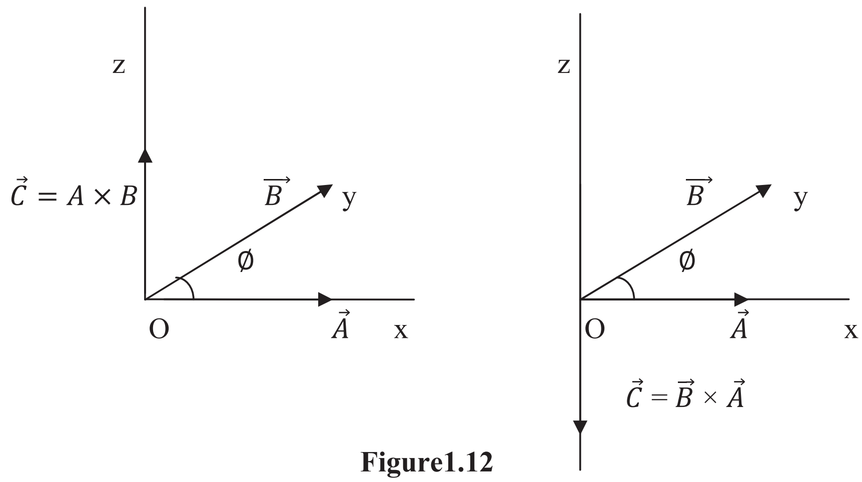

Figure1.12

Important properties of vector product

(i) Commutative law does not hold: From the definition of vector product of two vectors $\vec{A}$ and $\vec{B}$ the vector products are defined as

$$\vec{A} \times \vec{B} = \text{AB}\sin\emptyset\,\hat{n}$$

$$\vec{B} \times \vec{A} = \text{AB}\sin\emptyset\,(-\hat{n}) = -\text{AB}\sin\emptyset\,\hat{n} = -\vec{A} \times \vec{B}$$

Since in case of $\vec{B} \times \vec{A}$ the angle of rotation becomes opposite to case $\vec{A} \times \vec{B}$, hence product becomes negative.

Therefore, $\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$

(ii) Distributive law holds:
In case of vector product the distribution law holds.

$$\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$$

(iii) Product of equal vectors
If two vectors are equal then the angle between them is zero, and vector product becomes

$$\vec{A} \times \vec{A} = |A||A|sin\emptyset\,\hat{n} = 0$$

Hence the vector product of two equal vectors in always zero.

In case of Cartesian coordinate system if $\hat{\imath}, \hat{\jmath}, \hat{k}$are unit vectors along x, y and z axes then

$$\hat{\imath} \times \hat{\imath} = \hat{\jmath} \times \hat{\jmath} = \hat{k} \times \hat{k} = 0$$

(iv) Collinear vectors: Collinear vectors are vectors parallel to each other. The angles between collinear vectors are always zero therefore

$$\vec{A} \times \vec{B} = |A||B| sin\emptyset\, \hat{n} = 0$$

Thus two vectors are parallel or anti-parallel or collinear if its vector product is 0.

(v) Vector product of orthogonal vector : If two vectors $\vec{A}$ and $\vec{B}$ are orthogonal to each other then angle between such vectors is $\emptyset = 90°$ therefore

$$\vec{A} \times \vec{B} = AB\ sin\emptyset\, \hat{n}$$
$$\vec{A} \times \vec{B} = |A||B|\, \hat{n}$$

In Cartesian coordinate system if $\hat{\imath}, \hat{\jmath}, \hat{k}$ are unit vector along x, y and z axes then

$$\boldsymbol{\hat{\imath} \times \hat{\jmath} = \hat{k} \quad \hat{\jmath} \times \hat{k} = \hat{\imath}\ and\ \hat{k} \times \hat{\imath} = \hat{\jmath}}$$
$$\hat{\jmath} \times \hat{\imath} = -\hat{k} \quad \hat{k} \times \hat{\jmath} = -\hat{\imath}\ and\ \hat{\imath} \times \hat{k} = \hat{\jmath}$$

(vi) Determinant form of vector product: If $\vec{A}$and $\vec{B}$ are two vectors given as

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$$
$$\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}$$

Then

$$\vec{A} \times \vec{B} = \left(A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}\right) \times (B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k})$$
$$= A_xB_x\hat{\imath} \times \hat{\imath} + A_xB_y\hat{\imath} \times \hat{\jmath} + A_xB_z\hat{\imath} \times \hat{k} + A_yB_x\hat{\jmath} \times \hat{\imath} + A_yB_y\hat{\jmath} \times \hat{\jmath} +$$
$$+A_yB_z\hat{\jmath} \times \hat{k} + A_zB_x\hat{k} \times \hat{\imath} + A_zB_y\hat{k} \times \hat{\jmath} + A_zB_z\hat{k} \times \hat{k}$$

$$= A_xB_y\hat{k} - A_xB_z\hat{\jmath} - A_yB_x\hat{k} + A_yB_x\hat{\imath} + A_zB_x\hat{\jmath} - A_zB_y\hat{\imath}$$

(Since $\hat{\imath} \times \hat{\imath} = \hat{\jmath} \times \hat{\jmath} = \hat{k} \times \hat{k} = 0\ and\ \hat{\imath} \times \hat{k} = -\hat{\jmath}, \hat{\jmath} \times \hat{\imath} = -\hat{k}, \hat{k} \times \hat{\jmath} = -\hat{\imath}$)

$$= \hat{\imath}(A_yB_z - A_zB_y) - \hat{\jmath}\,(A_xB_z - A_zB_x) + \ k(A_xB_y - A_yB_x)$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Physical significance of vector product:

In physics, numbers of physical quantities are defined in terms of vector products. Some basic examples are illustrated below.

(i) **Torque:** Torque or moment of force is define as

$$\vec{\tau} = \vec{r} \times \vec{f}$$

Where $\vec{\tau}$ is torque, $\vec{r}$ is position vector of a point P where the force $\vec{f}$is applied. (Figure 1.13)

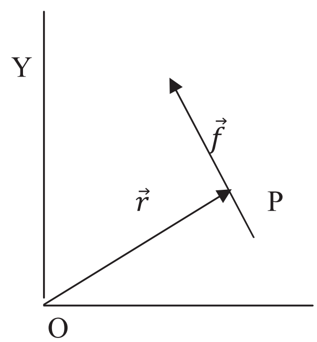

Figure 1.13

(ii) **Lorentz force on a moving charge in magnetic field**: if a charge q is moving in a magnetic field $\vec{B}$with a velocity $\vec{V}$at an angle with the direction of magnetic field then force $\vec{F}$experienced by the charged particle is give as;

$$\vec{F} = q(\vec{V} \times \vec{B})$$

This force is called Lorentz force and its direction is perpendicular to the direction of both velocity and magnetic field B.

(iii) **Angular Momentum**: Angular momentum is define as the moment of the momentum and given as:

$$\vec{L} = \vec{r} \times \vec{p}$$

Where $\vec{r}$ is the radial vector of circular motion and $\vec{p}$ is the linear moment of the body under circular motion, and $\vec{L}$ is angular momentum along the direction perpendicular to both $\vec{r}$ and $\vec{p}$. The law of conservation of angular momentum is a significant property in all circular motions.

1.3.3. Product of three vectors:

If we consider three vectors$\vec{A}$, $\vec{B}and$ $\vec{C}$, we can define two types of triple products known as scalar triple product and vector triple product.

1.3.3.1 Scalar Triple product:

Let us consider three vectors $\vec{A}$, $\vec{B}$and $\vec{C}$ then the scalar triple product of these three vectors is defined as $\vec{A}.(\vec{B}\times\vec{C})$ and denoted as $[\vec{A}\vec{B}\vec{C}]$. This is a scalar quantity. If we consider $\vec{A}$, $\vec{B}$ and $\vec{C}$ the three sides of a parallelopiped as shown in figure 1.14 then $\vec{B}\times\vec{C}$ is a vector which represents the area of parallelogram OBDC which is the base of the parallelogram. The direction of $\vec{B}\times\vec{C}$ is naturally along Z axis (perpendicular to both $\vec{B}$ and $\vec{C}$). If $\emptyset$ is the angle between the direction of vectors $(\vec{B}\times\vec{C})$ and vector $\vec{A}$, then the dot product of vectors $(\vec{B}\times\vec{C})$ and vector $\vec{A}$ is given as (figure 1.14)

$$\vec{A}\,.(\vec{B}\times\vec{C}) = |A||\vec{B}\times\vec{C}|cos\emptyset = ACos\emptyset(\vec{B}\times\vec{C}) = h.(\vec{B}\times\vec{C})$$

= Vertical height of parallelogram × area of base of parallelogram

= Volume of parallelogram = $[A\ B\ C]$.

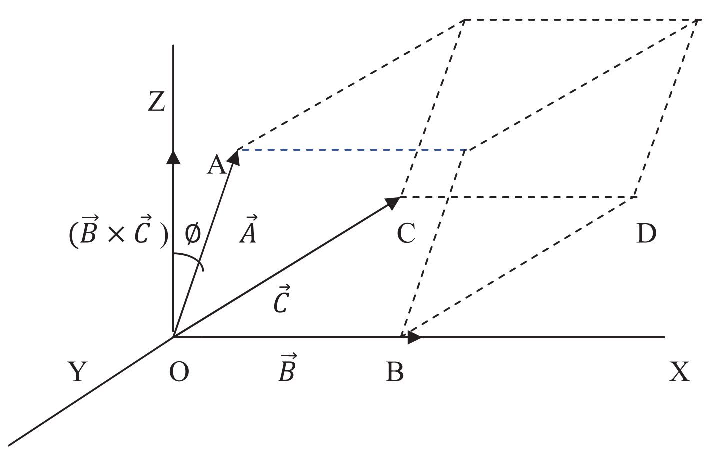

Figure 1.14

Therefore, it is clear that $\vec{A}\,.(\vec{B}\times\vec{C})$ represents the volume of parallelepiped constructed by vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ as its sides. Further, it is a scalar quantity as volume is scalar. It can also be noted that in case of scalar triple product the final product (volume of parallelepiped) remains same if the position of $\vec{A}, \vec{B}\ and\ \vec{C}$ or dot and cross are interchanged.

$$[\vec{A}\,\vec{B}\,\vec{C}] = \vec{A}.(\vec{B}\times\vec{C}) = \vec{B}.(\vec{C}\times\vec{A}) = \vec{C}.(\vec{A}\times\vec{B}) = (\vec{B}\times\vec{C}).\vec{A} = (\vec{C}\times\vec{A}).\vec{B} = (\vec{A}\times\vec{B}).\vec{C}$$

Scalar triple product can also be explained by determinant as

$$[\vec{A}\,\vec{B}\,\vec{C}] = \vec{A}.(\vec{B}\times\vec{C}) = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}$$

In case of three vectors to be coplanar, it is not possible to construct a parallelepiped by using such three vectors as its sides; therefore the scalar triple product must be zero.

$$[\vec{A}\,\vec{B}\,\vec{C}] = \vec{A}.(\vec{B}\times\vec{C}) = 0$$

1.3.3.2 Vector triple product:

The vector triple product of three vectors is define as

$$\vec{A}\times(\vec{B}\times\vec{C}) = (\vec{A}.\vec{C})\vec{B} - (\vec{A}.\vec{B})\vec{C}$$

The vector triple product is product of a vector with the product of two another vectors. The vector triple product can be evaluated by determinant method as given below.

$$(\vec{B}\times\vec{C}) = \begin{vmatrix} i & j & k \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}$$

$$= i(B_yC_z - B_zC_y) - j(B_xC_z - B_zC_x) + k(B_xC_y - B_yC_x)$$

$$\vec{A}\times(\vec{B}\times\vec{C}) = \begin{vmatrix} i & j & k \\ A_x & A_y & A_z \\ B_yC_z - B_zC_y & B_zC_x - B_xC_z & B_xC_y - B_yC_x \end{vmatrix}$$

$$= (\vec{A}.\vec{C})\vec{B} - (\vec{A}.\vec{B})\vec{C}$$

As in cross product the vector $\vec{A}\times(\vec{B}\times\vec{C})$ will be perpendicular to plane containing vectors $\vec{A}$ and $(\vec{B}\times\vec{C})$. Since $(\vec{B}\times\vec{C})$ is itself in the direction perpendicular to plane containing $\vec{B}$ and $\vec{C}$, therefore the direction of $\vec{A}\times(\vec{B}\times\vec{C})$ will be along the plan containing $\vec{B}$ and $\vec{C}$, $hence\ is\ a\ linear$ combination of $\overrightarrow{B}\ and\ \overrightarrow{C}$.

1.4 Summary

1. Physical quantities are of two types, scalar and vector. The scalar quantities have magnitude only but no direction. The vector quantities have magnitude as well as direction.
2. Two vector quantities can be added with parallelogram law and triangle law. In parallelogram law, the resultant is denoted by the diagonal of parallelogram whose adjacent sides are represented by two vectors. In triangle law, we place the tail of second vector on the head of first vector, and resultant is obtained by a vector whose head is at the head of second vector and tail is at the tail of first vector.
3. For subtraction, we reverse the direction of second vector and add it with first vector.
4. In case of more than two vectors we simply use Polygon law of vector addition.
5. Any vector can be resolved into two or more components. By adding all components we can find the final vector.
6. If a vector makes angles α, β and γ with three mutual perpendicular axes x, y and z respectively then $cos\,\alpha$, $cos\,\beta$ and $cos\,\gamma$ are called direction cosines.
7. Scalar product of two vectors is defined as $\overrightarrow{P}.\,\vec{Q} = PQ\cos\theta$ which is a scalar quantity.
8. Vector product of two vectors is defined as $\vec{A} \times \vec{B} = ABsin\emptyset\,\hat{n}$ which is a vector quantity. The direction of vector is perpendicular to $\vec{A}$ and $\vec{B}$.
9. If two vectors are parallel to each other then they are said to be collinear. For collinear vectors $\overrightarrow{P}.\,\vec{Q} = PQ$ or $\vec{P} \times \vec{Q} = 0$
10. If the angle between two vectors is 90^0, then vectors are called orthogonal. In this case $\overrightarrow{P}.\,\vec{Q} = 0$
11. Cross product of two vectors can also be calculated by determinant. The determinant form of cross product is

$$\vec{A} \times \vec{B} = \begin{vmatrix} i & j & k \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

12. Scalar triple product of three vectors can also be calculated by determinant. The determinant form of Scalar triple product is

$$\overrightarrow{A}.(\vec{B} \times \vec{C}) = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}$$

13. Vector triple product is defined as

$$\vec{A} \times (\vec{B} \times \vec{C}) = (\vec{A}.\vec{C})\vec{B} - (\vec{A}.\vec{B})\vec{C}$$