LECTURE NOTES

ON

APPLIED PHYSICS

I B. Tech I semester

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SYLLABUS:

UNIT – I DIELECTRIC AND MAGNETIC PROPERTIES: Dielectric Properties: Basic definitions, electronic, ionic and orientation polarizations-qualitative; Internal field in solids. Magnetic Properties: Basic definitions, origin of magnetic moment, Bohr magneton, classification of dia, para and ferro magnetic materials on the basis of magnetic moment, domain theory of ferro magnetism on the basis of hysteresis curve.

UNIT – II ACOUSTICS AND ULTRASONICS: Acoustics: Reverberation, reverberation time, Sabine's formula (qualitative), absorption coefficient, measurement of absorption coefficient, factors affecting acoustics of an auditorium and their remedies; Ultrasonics: Introduction; Generation of ultrasonic waves; Magnetostriction method, piezoelectric method, properties, applications.

UNIT – III EQUILIBRIUM OF SYSTEM OF FORCES: Introduction, basic concepts, system of forces, coplanar concurrent forces, force systems in space, parallel forces in plane; Force systems in space, couples, resultant, Lami's theorem, triangle law of forces, polygon law of forces, condition of equilibrium.

UNIT – IV FRICTION: Friction: Types of friction, limiting friction, laws of friction, angle of repose, equilibrium of body laying on rough inclined plane, Application of friction: ladder friction, wedge friction, screw friction.

UNIT – V DYNAMICS OF RIGID BODIES - MOMENT OF INERTIA: Rotational motion, torque, angular momentum, relation between torque and angular momentum, angular momentum of system of particles, moment of inertia, expression for moment of inertia, radius of gyration, theorems on moment of inertia, moment of inertia of thin rod, rectangular lamina, circular disc.

Text books:

1. Dr. K. Vijaya Kumar, Dr. S. Chandralingam, “Modern Engineering Physics”, Chand & Co. New Delhi, 1st Edition, 2010.
2. R. C Hibbler, "Engineering mechanics", Prentice Hall, 12th Edition, 2009.

Reference books:

1. R. K. Gaur, S. L. Gupta, “Engineering Physics”, Dhanpat Rai Publications, 8th Edition, 2001.
2. Timoshenko, D. H. Young, "Engineering mechanics", Tata Mc Graw Hill, 5th Edition, 2013.
3. Hitendra K Malik, A. K. Singh, “Engineering Physics”, Mc Graw Hill Education, 1st Edition, 2009.
4. S. S. Bhavikatti, "A text book of Engineering mechanics", New age international, 1st Edition, 2012.

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UNIT-I

Chapter 1. DIELECTRIC PROPERTIES

1.1. Introduction

- Dielectrics are insulating materials. In dielectrics, all electrons are bound to their parent molecules and there are no free charges.
- Even with normal voltage or thermal energy electrons are not released.
- Dielectrics are non metallic materials of high specific resistance and have negative temperature coefficient of resistance.
- Dielectrics are electrical insulators. They possess high resistivity values within the range 10^6 Ωm to 10^{16} Ωm. Under high voltage bias, they allow very little current. They with stand for very high voltages. The conduction is mostly associated with ionic motion through defects or hopping of charges. They have no free charges. The electrical properties of a dielectric are associated with inherent property of possessing electric dipoles.
- Dielectrics are the materials having electric dipole moment permanently or temporarily by applying electric field. These are mainly used to store electrical energy and as electrical insulators. All dielectrics are electrical insulators. But all electrical insulators need not be dielectrics. For example the vacuum is a perfect insulator. But it is not a dielectric. The study of dielectrics is essentially study of insulators.

1.2. Basic Definitions

Electric dipole

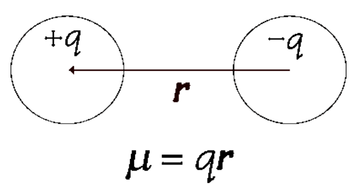


Fig. 1.1: Electric dipole

Two equal and opposite charges separated by a distance ‘r’ constitute a dipole.

Electric dipole moment (μ)

The product of charge and distance between two charges is called electric dipole moment.

$$\mu = q \times r$$

Units: coulomb - meter or Debye.

1 Debye = 3.33×10^{-30} coulomb - meter

Non-polar dielectrics

Mono atomic materials are made up of atoms. The centre of gravity of negative charge and the centre of gravity of positive charge of an atom coincide. That means even though there are two equal and opposite charges are not separated. Their dipole moment is zero.

$$\because \ \mu = q \times r = q \times 0 = 0$$

Such dielectrics are called Non- polar dielectrics.

Non-Polar Molecule

- Consider an atom. The positive charge of nucleus may be concentrated at a single point called as centre of gravity of the positive charge.
- The negative charge of electrons may be supposed to be concentrated at a single point called as Center of gravity of the positive charge.
- When the two centre of gravity coincide, the molecule is known as Non-polar molecule. The Non-polar molecules have symmetrical structure and zero electric dipole moment.
- Examples: H_2, N_2, O_2, CO_2, Benzene.

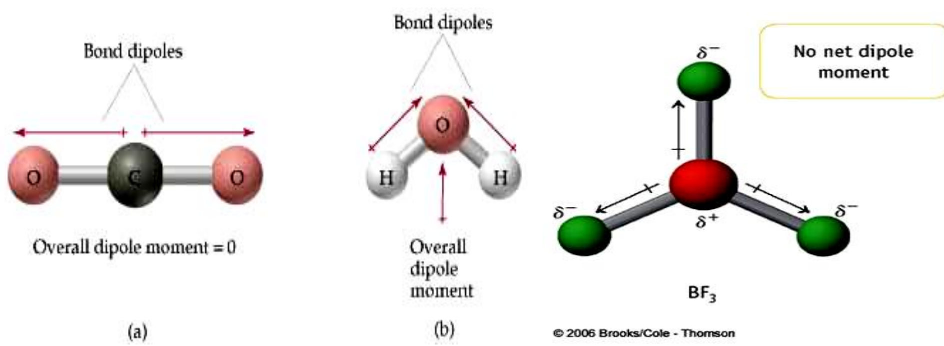


*Fig. 1.2: Non polar molecules with zero dipole moment (**μ=0**)*

- The electric dipole moment has a direction from positive charge to negative charge.

Polar dielectrics

- In polyatomic molecules, the center of gravity of negative charge distribution may not coincide with the center of positive charge distributions.
- There is an effective separation between centers of negative and positive charge distributions.
- The molecule has a net dipole moment. Such dielectrics are called Polar dielectrics.

Polar Molecules

They have unsymmetrical structure and have a permanent electric dipole moment. The Center of gravity of positive and negative charges do not coincide, the molecule is called as polar molecule.

e.g.:- H_2O, HCl, CO, N_2, NH_3 etc.

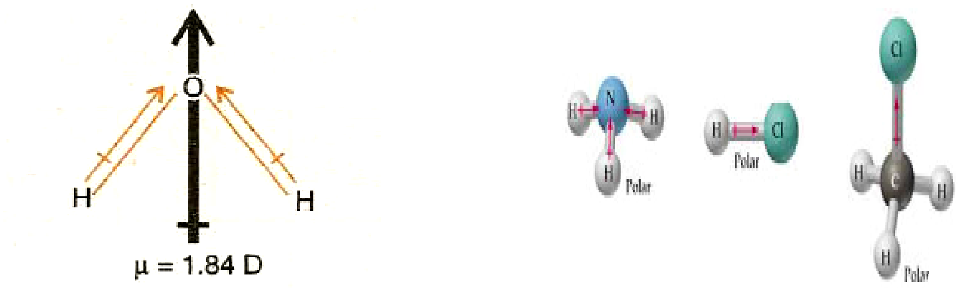


*Fig.1.3: Polar molecules with net dipole moment (**$\mu \neq 0$**)*

Dielectric constant ϵ_r (or) Relative permittivity of the medium

- It is the ratio between the permittivity of the medium and the permittivity of free space.

$$\epsilon_r = \frac{\epsilon}{\epsilon_0} \quad . \quad \text{It has no units.}$$

- Єr is also called as relative permittivity of the medium. It is a measure of polarization in the dielectric material.

$\epsilon_r = \epsilon_0 \epsilon_r$ where Є = absolute permittivity of the medium

Where Єo = permittivity of free space=8.854 x 10^{-12} F/m

Electric Polarization

When a dielectric substance is placed in an electric field, then positive and negative charges are displaced in opposite direction.

- The displacement of charges produces local dipoles.
- This process of producing dipoles by the influence of electric field is called electric polarization.

$$\text{dielectric Polarization} = P = \frac{electric\ dipolemoment}{volume}$$

$$P = \frac{\mu}{V}$$

$$P = \frac{q \times l}{A \times l} = \frac{q}{A}$$

$$P = \frac{\text{charge}}{\text{area}} = \text{surface charge density ‘}\sigma\text{’}$$

Polarizability (α)

The average dipole moment $\bar{\mu}$ is directly proportional to the electric field (E) applied.

$$\mu \ \alpha \ E$$

$$\mu = \propto E$$

$$\propto = \text{Polarizability} = \frac{\mu}{E} \text{ Farad/m}^2$$

Polarization Vector ($\vec{P}$)

It is defined as the average dipole moment per unit volume of a dielectric. If 'N' molecules are present per unit volume,

Then polarization vector $\vec{P} = \frac{N\bar{\mu}}{volume}$

$$\vec{P} = N\bar{\mu} \quad \text{coulomb/m}^2$$

Electric flux density (or) Electric displacement (D)

The number of electric lines of forces received by unit area is called Electric flux density.

$$D \; \alpha \; E$$
$$D = \epsilon E$$
$$\text{But } \epsilon = \epsilon_0 \, \epsilon_r$$
$$D = \epsilon_0 \, \epsilon_r \, E$$

Relation between Polarization (P), Electric field (E) & Dielectric constant ϵ_r

We know that electric flux density 'D' is written as

$$D = \epsilon \, E \qquad \rightarrow (1)$$

Where ϵ= absolute permittivity of medium

E = electric field

Where $E = \epsilon_0 \, \epsilon_r \qquad \rightarrow (2)$

Hence substitute (2) in (1)

$$D = \epsilon_0 \, \epsilon_r \qquad \rightarrow (3)$$

Where ϵ_0 = permittivity of free space = 8.854 $\times 10^{-12}$ F/m

ϵ_r = Dielectric constant or relative permittivity of the medium.

If 'P' is the polarization of the dielectric material due to the applied electric field (E), then the flux density 'D' is equal to the flux density in vacuum plus polarization of the material.

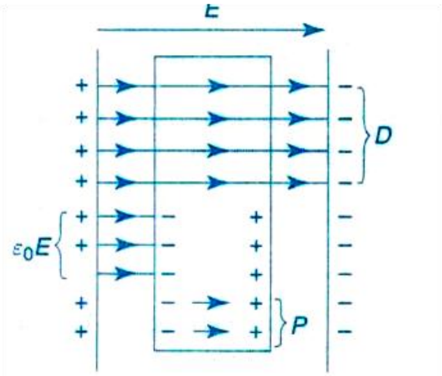


Fig. 1.4: Electric lines of forces in a polar dielectric

$\therefore D = \epsilon_0 E + P$ ` $\rightarrow (4)$

equating equations (3)& (4)

$\epsilon_0 \epsilon_r E = \epsilon_0 E + P$

$P = \epsilon_0 E [\epsilon_r - 1]$

$P = \epsilon_0 \epsilon_r E - \epsilon_0 E$ $\rightarrow (5)$

Put $[\epsilon_r - 1] = \chi$ $\rightarrow (6)$

Where χ = electric susceptibility

From equation (5) & (6)

$\Longrightarrow P = \epsilon_0 E \chi$

$\chi = \frac{P}{\epsilon_0 E}$ $\rightarrow (7)$

Conclusion:

$\Rightarrow P = E [\epsilon_r - 1]$

$\chi = \epsilon_r - 1$

$\chi = \frac{P}{\epsilon_0 E}$

Electric Susceptibility (χ)

The electric susceptibility 'χ' is defined as the ratio of polarization vector to the applied electric field 'E'.

$\chi = P/\epsilon_0 E$

χ has no units.

$\Rightarrow P = \chi \epsilon_0 E$

Where $\chi = \epsilon_r - 1$

Dielectric strength

It is defined as the minimum voltage required producing dielectric break down. Dielectric strength decreases with rising of temperature, humidity and age of the material.

1.3 Non-Polar Dielectric in an electric field

When a dielectric is placed in an electric field, say between the plates of a charged Condenser; the positive and negative charges are re oriented i.e. the center of gravity of positive charges is pulled towards the negative plate of the condenser and vice versa. Thus the net effect of the applied field is to separate the positive charges from the negative charges. This is known as Polarization of dielectric. The dielectrics which are polarized only when they are placed in an Electric field are called Non-polar dielectrics.

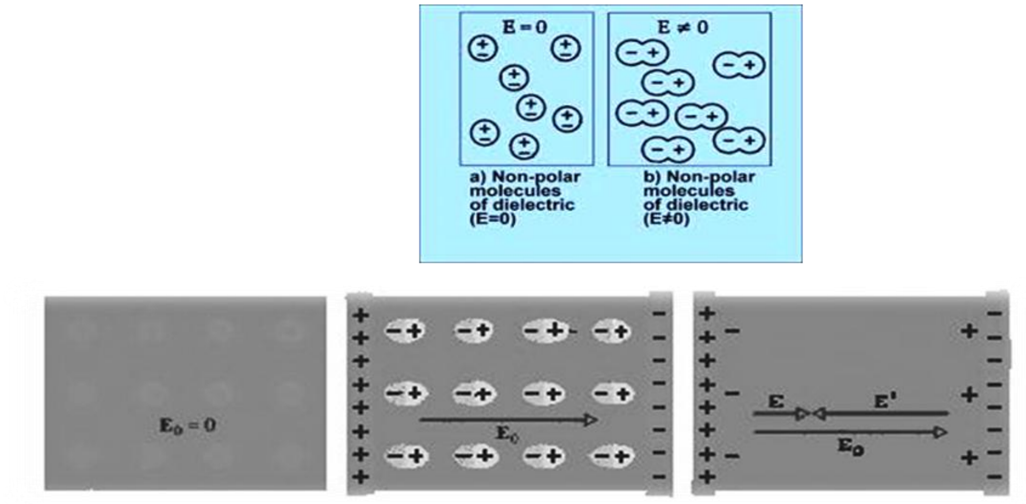


Fig. 1.5: (a) non polar dielectric in E=0 (b) Non polar dielectric when 'E' is applied (c) Totally polarized non polar dielectric with net field

Thus if the dielectric is placed in an electric field, induced surface charges appear which tend to weaken the original field with in the dielectric. That means E^1 opposes the original field E_o.

1.4 Polar dielectric in electric field

We know that polar dielectric have permanent dipole moments with their random orientations. In the presence of an electric field, the partial alignment of dipoles takes place.

Polar dielectrics already possess some dipole moment inside due to the presence of permanent atomic dipoles. But these are randomly oriented when no field is applied.

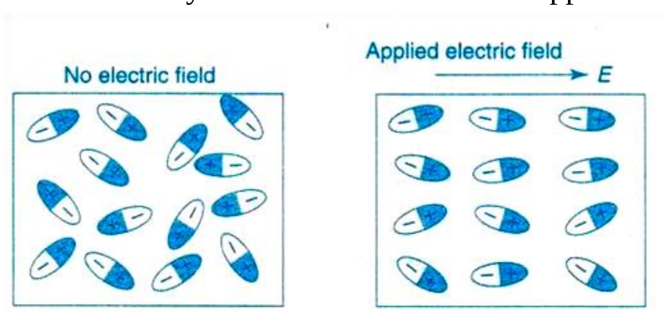


Fig. 1.6: Polar dielectric orientation without field and with field.

Polar dielectrics already possess some dipole moment inside due to the presence of permanent atomic dipoles. But these are randomly oriented when no field is applied. Their dipole moment and polarization increases since dipoles align along the field direction gives some extra polarization. Hence

$$P = \left\{ \begin{array}{l} \text{Polarization already} \\ \text{Existing due to permanent} \\ \text{dipoles} \end{array} \right\} + \left\{ \begin{array}{l} \text{Polarization induced} \\ \text{due to electric} \\ \text{field} \end{array} \right\}$$

$$P = P_p + P_i$$

1.5. The Local field (or) Internal field E_i (or) E_{local}

Definition: In dielectric solids, the atoms or molecules experienced not only the external applied electric field but also the electric field produced by the dipoles. Thus the resultant electric field acting on the atoms or molecules of dielectric substance is called the "Local field or an internal field."

Derivation:

Consider a dielectric material placed in an External field 'E_1', placed between the parallel plates of a capacitor. As a result opposite type of charges are induced on the surface of dielectric.

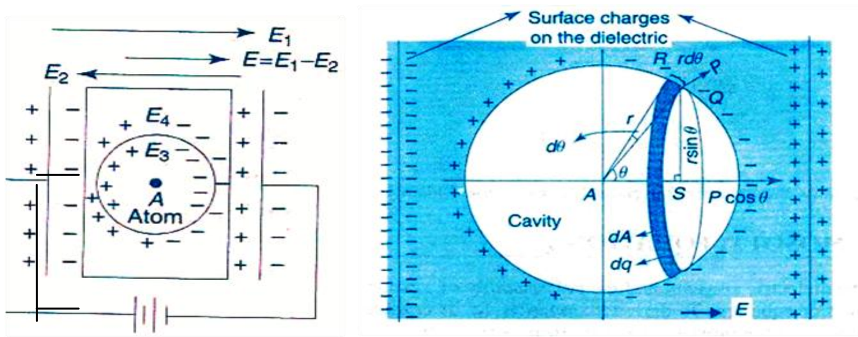


Fig. 1.7 (a) Polar dielectric in electric field (b) Enlarged view of spherical cavity

Imagine a small spherical cavity of radius 'r'. In this sphere inside dipoles are present. Consider a dipole at the center of spherical cavity. This dipole experiences the following fields, in addition to the externally applied field 'E_1'.

The total internal field experienced by the dipole

$$E_{local} = E_i = E_1 + E_2 + E_3 + E_4 \qquad \rightarrow (1)$$

Where E_1 = External applied field. Here,

(a) The field 'E_2' produced by induced charges on the dielectric sample near the surface.
(b) The field E_3 arising from dipoles inside the sphere. E_3 depends on crystal symmetry. [For isotropic materials E_3=0]
(c) The field E_4 is due to polarization of charges on the surface of spherical cavity. It is called the Lorentz cavity field.

The surface charge density on the surface of the spherical cavity is $P\cos\theta$.
If 'ds' is the area of the surface element shaded in figure shown.
Then charge on the surface element (q_1) is
= (normal component of polarization) x (area of the surface element)

$$q_1 = (P\cos\theta)(ds) \qquad \rightarrow (2)$$

Let a test charge q_2 = q placed at the center of cavity.

From coulombs' law, the force experienced between the surface charges.

$$dF = \frac{1}{4\pi} \frac{q_1 q_2}{r^2} dF = \frac{1}{4\pi\epsilon_0} \frac{(Pcos\theta ds).q}{r^2}$$

$$\frac{dF}{q} = \left[\frac{1}{4\pi\epsilon o}\right]\left[\frac{Pcos\theta ds}{r^2}\right] \qquad \rightarrow (3)$$

The resulting electric field $E_4 = \frac{dF}{q} = \left[\frac{1}{4\pi\epsilon o}\right]\left[\frac{Pcos\theta ds}{r^2}\right]$ → (4)

The electric field is resolved into two components:

One component is along the direction of 'P' & other perpendicular to it.

The Perpendicular components cancel themselves out leaving only the horizontal components. Hence the sum of all such horizontal components of electric field for the whole Surface is:

$$E_4 = \int dE_4 \cos\theta = \int \frac{1}{4\pi\epsilon o} \frac{(Pcos\theta)(cos\theta ds)}{r^2} \qquad \rightarrow (5)$$

The surface area of the ring $ds = 2\pi r^2 \sin\theta d\theta$ → (6)

Substitute (6) in (5)

$$E_4 = \frac{1}{4\pi\epsilon o} \int \frac{Pcos^2\theta 2\pi r^2 sin\theta d\theta}{r^2}$$

Limits are $\theta = 0$ to π

$$E_4 = \frac{P}{2\epsilon o} \int_0^\pi cos^2\theta d\theta \qquad \rightarrow (7)$$

Let $\cos\theta = z$

$-\sin d\theta = dz$ & Limits are $z = 1$ to $z = -1$

Equation (7) becomes $E_4 = \frac{P}{2\epsilon o} \int_1^{-1} z^2 (-dz)$

$$E_4 = \frac{-P}{2\epsilon 0} \int_{+1}^{-1} z^2 \, dz$$

$$= \frac{-P}{2\epsilon o}\left[\frac{z^3}{3}\right]_1^{-1} = \frac{+P}{3\epsilon o}$$

$$E_4 = \frac{P}{3\epsilon o} \qquad \rightarrow (8)$$

Substitute the value of E_4 in equation (1)

Total internal field (or) local field

$$E_i = E_{loc} = E_1 + E_2 + E_3 + E_4$$

Here $E_3 = 0$

$$\therefore E_i = E_1 + E_2 + E_4$$

Let $E_1 + E_2 = E$

$$\Longrightarrow \quad E_i = E + E_4$$

$$E_i = E + \frac{P}{3\epsilon_o} \qquad \rightarrow (9)$$

1.6 Types of Polarization

Polarization is the process of inducing dipole moment in a molecule. There are four types of polarization. They are:

(1) Electronic Polarization
(2) Ionic Polarization
(3) Orientation (or) Dipolar Polarization

(4) Space charge polarization

1.6.1. Electronic Polarization:

Definition:

When an electric field is applied on a dielectric material then all the positive nuclei of atoms move in the field direction and all the negative electron cloud of atoms move in opposite directions, hence dipoles are formed to produce dipole moment.

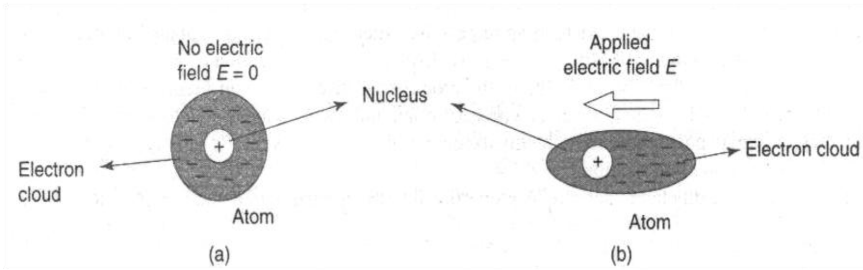


Fig. 1.8. (a) Un polarized atom in the absence of field
(b)Electronic polarization due to distortion of electron cloud by the field E

- The electron cloud readily shifts towards the positive end of the field. The extent of shift by electrons is proportional to field strength.
- Hence dipole moment is the product of charge and shift distance.

Expression for Electronic Polarizability $\boldsymbol{\alpha_e}$

Electronic Polarizability $\alpha_e = 4\pi\epsilon_0 R^3$ → (1)

Where R= radius of an atom.

1.6.2 Ionic Polarization

Ionic polarization takes place in ionic dielectrics due to displacement of positive and negative ions by the influence of external electric field.

Expression for Ionic Polarizability

When an electron field is applied on an ionic dielectric then positive ions move in the field direction & negative ions move in opposite direction, hence dipoles will be formed. This phenomenon is known as ionic polarization.

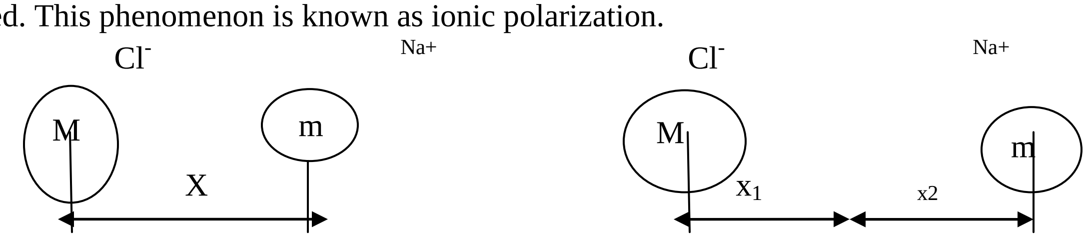


Fig. 1.9 (a) In the absence of field *Fig. 4.9(b) when field is applied*

Let 'e' the charge of ions and M and m be the masses of negative and positive ions.

$$\alpha_i = \frac{e^2}{\omega_0^2}\left[\frac{1}{M} + \frac{1}{m}\right] \quad \rightarrow (2)$$

Where, ω_0 = natural frequency of the ionic molecule.

1.6.3 Orientational Polarization

Definition:

When Electric field is applied on a polar dielectric then all the dipoles tend to rotate In the field direction, hence dipole moment increases gradually. This phenomenon is known as dipolar (or) orientational polarization.

Expression for Orientational (or) dipolar Polarisability

Orientation Polarisation takes place only in polar dielectrics in which dipoles orient in random manner such that the net dipole moment is zero. When Electric field is applied, all the dipoles try to rotate in the field direction as shown in the figure 5.10.

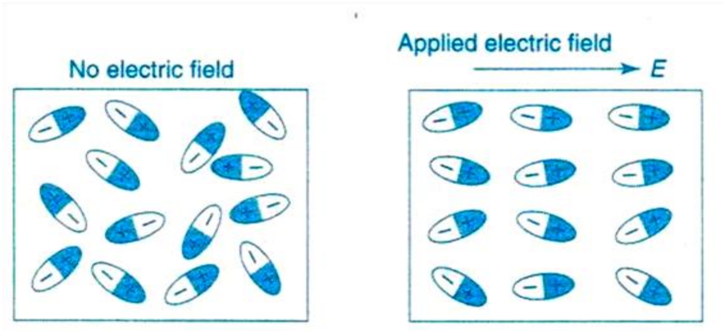


Fig. 1.10: Orientational polarization

Orientational (or) dipolar polarisability, $\alpha_0 = \frac{\mu^2}{3K_BT}$

Where, μ = dipole moment, K_B = Boltzmann constant and T = absolute temperature.

The total polarizability, $\alpha = \alpha_e + \alpha_i + \alpha_0$

$$\alpha = \{4\pi\epsilon_0 R^3\} + \left\{\frac{e^2}{\omega_0^2}\left[\frac{1}{M} + \frac{1}{M}\right]\right\} + \left\{\frac{\mu^2}{3K_BT}\right\}$$

The total polarization, P = N α E

$$= N\,\left[\,\{4\pi\epsilon_0 R^3\} + \left\{\frac{e^2}{\omega_0^2}\left[\frac{1}{M} + \frac{1}{M}\right]\right\} + \left\{\frac{\mu^2}{3K_BT}\right\}\right] E$$

Chapter-2 MAGNETIC PROPERTIES

2.1 Introduction

- The magnetic effects in magnetic materials are due to atomic magnetic dipoles in the materials. These dipoles result from effective current loops of electrons in atomic orbits, from effects of electron spin & from the magnetic moments of atomic nuclei.
- The electric currents in an atom are caused by orbital and spin motions of electrons and those of its nucleus. Since all these motions of charged particles form closed electric currents, they are equivalent to "magnetic dipoles". When such dipoles are subjected to an external electric field, they experience a torque which tends to align their magnetic moments in the direction of the externally applied field.

2.2. Definitions

Magnetic dipole

Each tiny dimension of a magnetic material (or) atoms in magnetic materials is called magnetic dipole. This magnetic dipole produces magnetic moment depending on the alignment with respect to the applied magnetic field.

Magnetic flux (Φ)

It is defined as the amount of magnetic lines of forces passing perpendicularly through unit area of a given material. It is denoted by '**Φ**'

$$\mathbf{\Phi} = AB$$

Where A= Area of cross section of the material in m^2

B = magnetic Induction in Wb/ m^2

Units: Weber (Wb)

Intensity of Magnetization (M)

When a material is magnetized, it develops a net magnetic moment. The magnetic moment per unit volume is called Intensity of magnetization

$$\text{Magnetization (M)} = \frac{\text{Magnetic moment}}{\text{Volume}}$$

Units: Amp/m

Magnetic Induction (B)

Magnetic induction at a point is defined as the force experienced by a unit North Pole Placed at that point. It is denoted by 'B'

$$\text{i.e. } B = \frac{\Phi}{A} \text{ weber / m}^2$$

Magnetizing field strength (H)

When a medium is exposed to a magnetic field of intensity 'H', it causes an induction 'B' in the medium.

$$\text{i.e.}\quad B \propto H$$

$$B = \mu H$$

Where μ = absolute permeability of the medium.

If the medium is air or vacuum

$$B=\mu_0 H$$

$\mu_0 =$ permeability of free space i.e. air or vacuum

$\mu_0 = 4\pi \times 10^{-7}$ H/m

Units for H: Amp /m.

Permeability (μ)

It indicates, with which the material allows magnetic lines of force to pass through it.

Or

It is the ability of the medium to pass magnetic lines of forces through it.

There are three Permeabilities i.e. μ_1, μ_0, μ_r

$$\mu = \mu_0\ \mu_r$$

Where μ = Absolute permeability of the medium

μ_0 = Permeability of free space i.e. air or vaccum

μ_r = Relative permeability of the medium

Magnetic moment

Magnetic moment μ_m = (current) × (area of circulating orbit)

$\mu_m = (I) \times (\pi r^2)$

Units: Amp-m^2

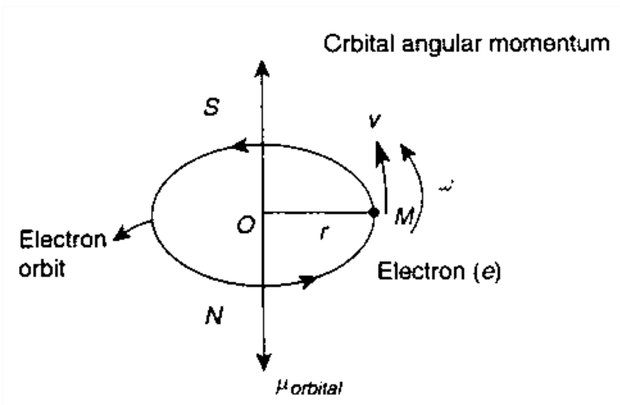


Fig.2.1 Revolving electron in an atom produces magnetic moment

When the magnetic dipoles (atoms consisting of charged particles like protons & neutrons) undergo orbital motion (or) spin motion produces a magnetic moment. Since motion of charged particles is considered as closed electric current loops which inturn produces a magnetic moment.

Magnetic susceptibility (χ)

If H is the applied magnetizing field intensity and M is the amount of magnetization of the material, Then $\chi = \frac{M}{H}$

$\chi = 0$ in vacuum

$\chi = +ve$ for paramagnetic and Ferro magnetic materials

$\chi = -ve$ for diamagnetic materials

Units:It has no units.

2.2.1 Relation between B, H, M

We know that

$$B = \mu H \qquad \rightarrow (1)$$

But $\mu = \mu_0 \mu_r$

$$B = \mu_0 \mu_r H \qquad \rightarrow (2)$$

Adding &subtracting with $\mu_0 H$ on right hand side of equation (2)

$$B = [\mu_0 \mu_r H] + \mu_0 H - \mu_0 H$$

$$= [\mu_0 \mu_r H - \mu_0 H] + \mu_0 H$$

$$= \mu_0 H [\mu_r - 1] + \mu_0 H$$

But $M = H [\mu_r - 1] \qquad \rightarrow (3)$

Now eq(3) becomes $B = \mu_0 M + \mu_0 H$

$$B = \mu_0 [H + M] \qquad \rightarrow (4)$$

Consider equation (3), $M = H [\mu_r - 1]$

$$\frac{M}{H} = \mu_r - 1 \qquad \rightarrow (5)$$

But magnetic susceptibility $\chi = \frac{M}{H}$

From equations (5) and (6)

$$\chi = \frac{M}{H} = \mu_r - 1$$

$$\mu_r = 1 + \chi$$

2.3. Origin of magnetic moment (Or) Sources of magnetic moment

In atoms, the permanent magnetic moment arises due to

a) Orbital motion of electrons and its magnetic moment is called orbit magnetic moment of electrons (μ_l)
b) The spin of electrons and its magnetic moment is called spin magnetic moment of electrons (μ_s)
c) The spin of nucleus (due to protons) and its magnetic moment is called spin magnetic moment of the nucleus. (μ_n or μ_p).

Explanation

a) Magnetic moment due to orbital motion of the electrons (μ_l)

Let us consider an electron of charge 'e' revolving around a nucleus in time period 'T' in a circular orbit of radius 'r'. Then a magnitude of circular current 'I' is given by

$$I = \frac{Charge}{Time} = \frac{e}{T} \quad \rightarrow (1)$$

But $\quad T = \frac{2\pi}{\omega}$

Where ω = angular velocity of electron

$$I = \frac{e\omega}{2\pi}$$

But magnetic moment of electron is $\mu_l = I \times A$

μ_l = current area of circulating orbit

$$\mu_l = \frac{e\omega}{2\pi}(\pi r^2)$$

$$\mu_l = \frac{e\omega r^2}{2} \quad \rightarrow (2)$$

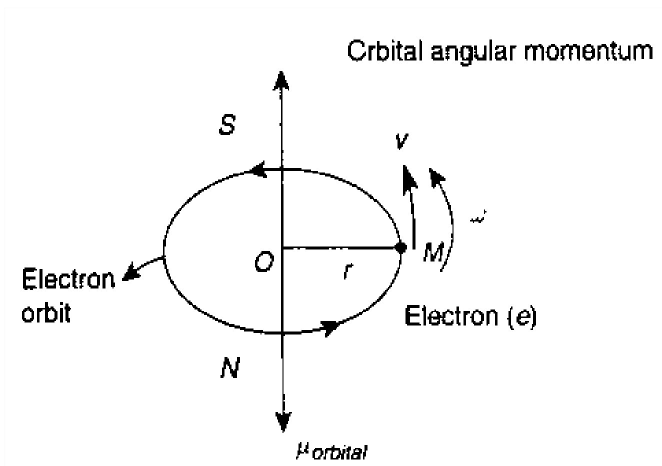


Fig. 2.2 Orbital magnetic moment of electrons

We know that angular momentum of any particle, L = mωr^2

Substituting eq.(4) in eq.(3) we get

Orbital magnetic moment, $\mu_l = (-\frac{e}{2m}).L \quad \rightarrow (5)$

[-ve sign indicates μ_l and are in opposite directions]

$$\mu_l = \quad (-\frac{e}{2m})\ L$$

But from Bohr's atomic model

$$mvr = \frac{nh}{2\pi}$$

$$L = \frac{lh}{2\pi} \quad \text{Where } l = \text{orbital quantum number}$$

L = orbital angular momentum

The values of l = 0, 1, 2---------- (n-1)

Hence $\mu_l = (-\frac{e}{2m})\,(\frac{lh}{2\pi})$

$$\mu_l = -\,(\frac{eh}{4\pi m})\,l \qquad \rightarrow (6)$$

Where $\frac{eh}{4\pi m} = \mu_B$ is a constant called Bohr magneton and its value is 9.27×10^{-24} amp-m^2

Hence eq(6) becomes

$$\mu_l = l\,\mu_B \qquad \rightarrow (7)$$

Bohr magneton is the fundamental unit of magnetic moment.

It is clear from eq (7) that electron can take only certain specified values of magnetic moment depending on the value of 'l'.

Bohr suggested that both magnitude and direction of 'l' are quantified. It is known as "Spatial quantization".

The spatial quantization introduces a new set of quantum numbers.

(a) Orbital magnetic quantum number (m_l)

(b) Spin magnetic quantum number (m_s)

For example: If electron is in'p' shell. Then n = 2, L= 0 to n-1, L = 0, 1,

If electron is placed in external magnetic field then eqn (6) can be written as

$$-\,(\frac{eh}{4\pi m})\,m_l \qquad \rightarrow (8)$$

Hence for'p' shell electron, m_l= 0 to ± L. The values are m_l = -1, 0, 1

Hence eqn (8) becomes

$$-\,(\frac{eh}{4\pi m}),\ 0,\ (\frac{eh}{4\pi m})$$

Therefore we have "Three" possible orientations for electron in d-shell which is shown in the figure 6.3.

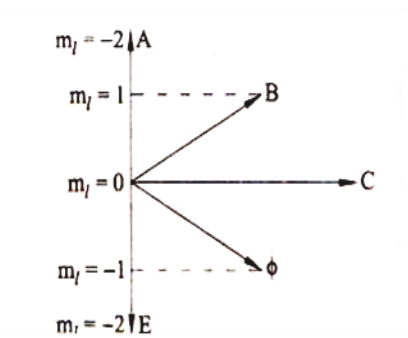


Fig. 2.3 The three possible orientations of electron

'OC' represents the orientation of electron if m_l = 0

'OB' represents the orientation of electron if m_l = +1

'OD' represents the orientation of electron if m_l = -1

b) Magnetic moment of electrons due to spin of electrons (μ_s)

According to quantum theory; electrons should have intrinsic angular momemtum due to spin. Spin is also quantized both in magnitude and direction spin can take only one value i.e ½ or - ½. The magnetic moment produced due to spin of electrons is called spin magnetic moment (μ_s).

It is given by

$$\text{Spin magnetic moment } \mu_s = -2\left(\frac{e}{2m}\right) S \qquad \rightarrow (9)$$

Where S=spin angular momentum, e = charge of electron, m = mass of electron

$$S = \frac{sh}{2\pi}$$

where S = spin quantum number

h = Planck's constant.

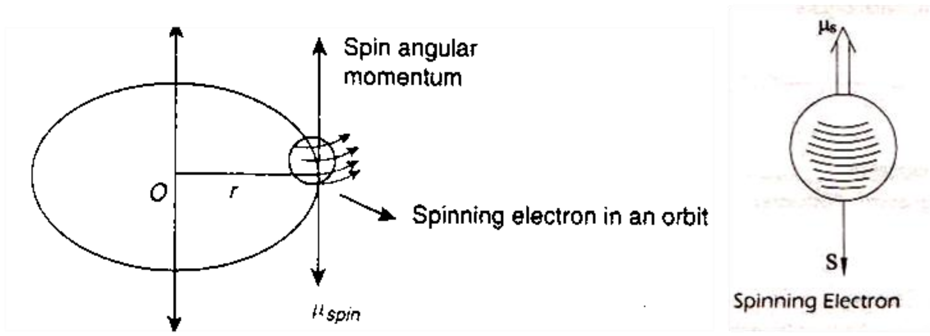


Fig. 2.4 Spin magnetic moment of electrons

From equation (9), $\mu_s = -2\left(\frac{e}{2m}\right) S$

Since $S = \frac{sh}{2\pi}$

$$\mu_s = -2\left(\frac{e}{2m}\right)\left(\frac{sh}{2\pi}\right)$$

$$s = \pm\frac{1}{2}, \quad \mu_s = \pm\frac{eh}{4\pi m}$$

$$\mu_s = \frac{eh}{4\pi m}, -\frac{eh}{4\pi m}$$

$$\mu_s = +\mu_{\boldsymbol{B}}, -\mu_{\boldsymbol{B}}$$

Hence spin magnetic moment of electron is equal to μ_B. That is one Bohr magneton

Hence there are two possible orientations of electron.

Conclusion: Para magnetism, Ferro magnetism is due to spin magnetic moment.

Diamagnetism is due to orbital magnetic moment.

(c) Magnetic moment due to Nuclear spin or spin of all protons (μ_n)

The magnetic moment of the nucleus is given by $\mu_n = \frac{eh}{4\pi m_p}$ $\rightarrow$ (10)

Where m_p= mass of proton

The constant $\frac{eh}{4\pi m_p}$ is called nuclear magneton.

The value of nuclear magneton $\frac{eh}{4\pi m_p} = 5\times10^{-27}$ A-m^2

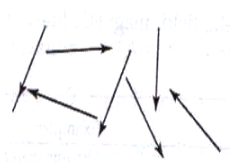
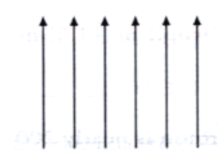
This is small when compared to Bohr magneton.

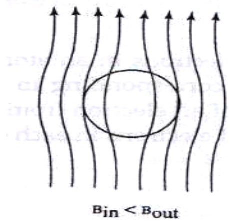
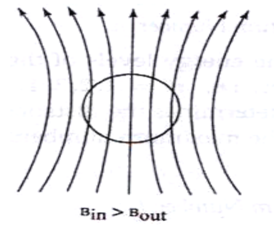
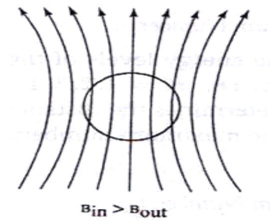
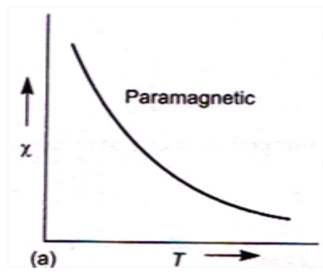
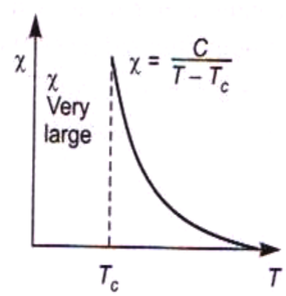
2.4. Classification of Magnetic Materials

Magnetic materials are classified as follows:

a) Diamagnetic
b) Paramagnetic
c) Ferro magnetic
d) Anti Ferro magnetic
e) Ferric magnetic or ferrites

Differences between various types of magnetic material

Diamagnetic materials	**Paramagnetic materials**	**Ferromagnetic materials**
1.Diamagnetism: It is the property of the material which has repulsive nature (or) opposing magnetization.	*1.Paramagnetism:* It is the property of the material which has weak attractive force.	*1.Ferromagnetism* It is property of the material which has strong attractive force.
2.The property is due to orbital motion of electrons	2.The property is due to spin of electrons	2.The property is due to spin of electrons
3.There is no spin	3.Spin is random 	3.Spin is parallel 
4. These materials are lack of magnetic dipoles	4.These materials have permanent dipoles	4.They have permanent magnetic dipoles
5. They do not possess permanent dipole magnetic moment (it is zero). Hence spontaneous magnetization is zero.	5. They possess permanent magnetic dipole moment. But there is no spontaneous magnetization in the absence of external field. Due to random spin.	5. They possess permanent magnetic dipole moment. Also in the absence of field they have spontaneous magnetization even in the absence of external field due to parallel

6. 	6. 	6. 
7.The relative permeability $\mu_r < 1$	7. The relative permeability $\mu_r > 1$.	7.The relative permeability $\mu_r >> 1$
8.Susceptibility χ is small and negative	8. Susceptibility is small but positive	8. Susceptibility is large and positive
9. χ does not depend on temperature. No particular is graph is drawn.	9. χ depends on temperature 	9. χ depends on temperature 
10. χ does not depend on temperature	10. $\chi = c/T$ (curie law) C=Curie constant T = absolute temperature	10. $\chi = c/T\text{-}\theta$ Curie-Weiss law θ = Curie temperature
11.Examples Cu, Au, Zn, H_20, Bi etc. organic materials.	11.Examples: Al, Pt, Mn,CuCl2 etc. Alkali & transition metals.	11.Examples: Fe, Ni, Co, MnO, $Fe_2\ 0_3$, Zn ferrite, Ni ferrite, Mn ferrite

2.5. Domain theory (or) Weiss theory of Ferromagnetism

- According to Weiss, Ferromagnetic material consists of a number of regions called "Domains" [~10^{-6} mts] which are spontaneously magnetized.

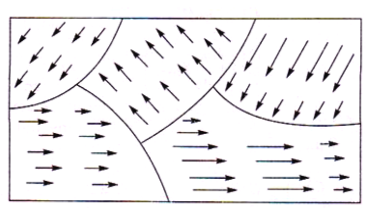


Fig. 25.5: Different possible orientations of domains

- Spontaneous magnetization is due to parallel alignment of all magnetic dipoles (in each domain) even when no external field is applied.
- Different domains possess different orientations hence net magnetization is zero.
- When an external field is applied there are two possible ways of alignment of domains. They are
  1. By motion of domain walls
  2. By rotation of domain walls
- Domains arise to minimize the energy of the material. The total internal energy is minimum. The alignment of domains, parallel to field is discussed as follows:

a) The domains which are parallel to the direction of applied magnetic field will grow in size than other domains. This is called "Motion of domain walls". Also other domains which are opposite to the field direction are reduced. This is shown in the figure 5.6.

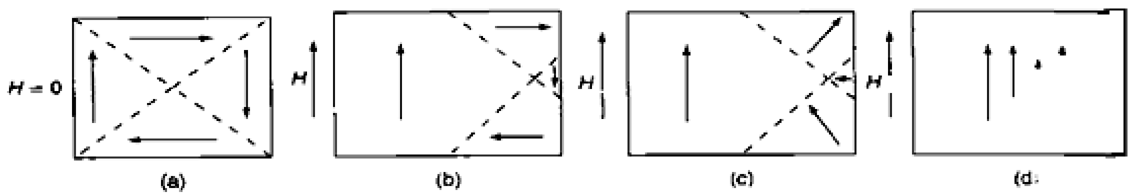


Fig. 2.6 (a) Domain orientation in the absence of the magnetic field (b) Domain enhancement shrinkage due to weak fields (c) Domain ratation due to strong fields (d) Saturation due to very high fields

b) As the magnetic field is strong, the magnetic moments of the domains can rotate in the applied field direction. This is called "rotation of domain walls".

2.5.6. Domain theory of ferromagnetism based on the basis of B-H curve (or) Hysteresis curve

Definition: Hysteresis means the lagging of magnetization "B" behind the applied magnetizing field "H". The energy supplied to the specimen during magnetization is not fully used. The balance of energy left in the material is produced as heat i.e. loss of heat called" Hysteresis Loss".

- This phenomenon of magnetic Hysteresis is an "Irreversible" characteristic of ferromagnetic material. The loop (or) area refers to the hysteresis loop. Hysteresis loss occurs in ferromagnetic materials below Curie temperature.

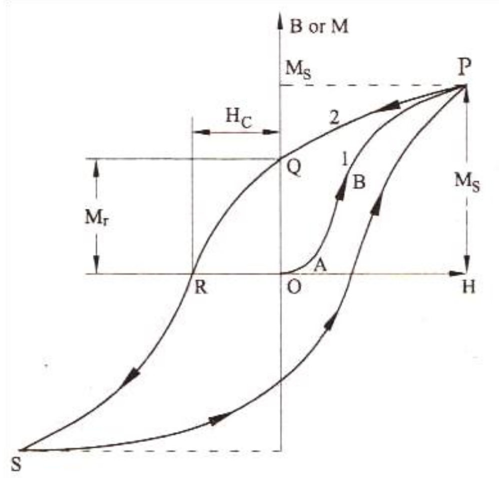


Fig. 2.7: Magnetic Hysteresis curve (or) B – H curves of a Ferro magnetic material

The complete cycle of operation is discussed as follows:

- When the magnetic field is applied on a ferromagnetic material the magnetization increases slowly and reaches a constant M_s called saturation magnetization
- In fig 5.7 from point O to A, the displacement of domain walls takes place. When the field is suddenly off, the domains again go for original position.
- From point A to B, as the field is further increased, the magnetization also increases. Here when the field is made off the domain displacement does not return back to original condition.
- For higher fields the magnetization reaches maximum ie saturation magnetization M_s due to rotation of domain walls.
- In this case at the region B to P, if the field is suddenly made off, the domains does not return back to original direction. But there is some magnetic field remained inside the specimen.
- The point M_s is called saturation of magnetization. When the field is off, the curve does not go back to 'O'[as shown in fig] but creates a new path to a point M_r called "retentivity" (or) residual (or) remanence Magnetization
- To reduce the residual magnetism to zero, a negative field 'Hc' has to be applied. When the sufficient negative field is applied, the residual; magnetization becomes zero and this field is known as "negative coercive field" ($-H_c$) or coercively. Further again if the negative field is applied then magnetization increases but in negative direction. This is known as negative saturation magnetization ($-M_S$).
- If the negative field is decreased back to zero the negative saturation of magnetization will not reach the initial path at '0' but creates a new path and reaches a point called negative residual magnetism '$-M_r$' know as negative receptivity.
- To decrease the negative residual magnetism to zero some positive field is applied. The amount of magnetic field required to bring residual magnetization to zero is known as positive coercive field (H_c).

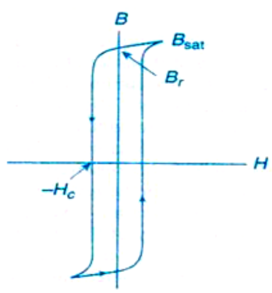
- Further the increase of positive magnetic field the magnetization reaches again to positive saturation (M_s) and this is a cyclic process.
- The final conclusion is that when the magnetization vector is started from origin 'O' will not reach back to that point. "The magnetization lags behind H". This is called magnetic hysteresis loss measured in the area of the loop (or) curve. If the loop area of a ferromagnetic material is large, more energy is wasted. This is also called as "dielectric loss" for one complete cyclic operation.

2.7. Differences between soft and hard magnetic materials

Based on area of the hysteresis loop, they are classified as soft and hard magnetic Materials.

Soft magnetic materials

1. They have small hysteresis loss due to small area of hysteresis loop

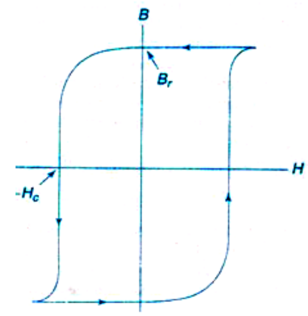


Hysteresis loop of a soft magnetic material

2. The domain wall movement is easier
3. The coercively ($-H_c$)and retentively (M_r) are small
4. They can be easily magnetized and demagnetized
5. The value of permeability and susceptibility are large
6. Magnetization energy is small
7. They are used as temporary magnetic or electro magnetic
8. Examples: ferrites, Fe-si alloy, Fe-Ni alloy
9. In magnetic inductors, cores, storage of data, switching circuits, audio frequency applications, magnetic amplifier etc as electro magnets.

Hard magnetic materials

1. They have large hysteresis loss due to large area of hysteresis loop.



2. The domain wall movement is difficult due to presence of impurities crystal imperfections
3. The coercively($-H_c$) and retentively (M_r) are large
4. They cannot be easily magnetized and demagnetized
5. The values of permeability and susceptibility are small
6. Magnetostriction energy is large
7. They are used as permanent magnets
8. Example: Al-Ni-Co, Cu-Ni-Fe, Cu-Ni-Co etc
9. For production of permanent magnets which are used in magnetic detectors, microphones flux meters, voltage regulators etc.

UNIT-II

Chapter-3 ACOUSTICS OF BUILDINGS

3.1 Introduction:

The branch of physics that deals with the process of generation, propagation and hearing (reception) of sound in a room, be it a small room or an auditorium is called acoustics. Architectural acoustics, also called acoustics of buildings, deals with the behavior of sound waves in a closed space. It deals with the design and construction of acoustically good buildings, music halls, recording rooms and movie theatres, where the audience receives the best sound quality.

3.2 Reverberation and time of Reverberation:

A sound produced inside a hall will propagate in all directions. Sound waves incident on the surfaces of walls, floor, ceiling and furniture inside a hall, will be multiply reflected. As the source of sound is turned off, the listener hears the sound with gradually reducing intensity for some time due to the persistence of sound by multiple reflections at different places in the room.

A listener inside the hall will receive the sound waves directly from the source, as well as the reflected waves. *The persistence of audible sound even after the source of sound is turned off is called Reverberation. The time taken by the sound intensity to fall to one millionth* $\left(\frac{1}{10^6}\right)$ *of its Initial intensity i.e., the intensity just before the source of sound is turned off is called Reverberation time.*

3.3 Sabine's formula for reverberation time:

According to Sabine's law, the reverberation time T is seconds is expressed as,

$$T = \frac{0.165V}{\sum aS}$$

Where, V is the volume of the hall in m^3 and $\sum aS$ is given by,

$$\sum aS = a_1S_1 + a_2S_2 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots + a_nS_n$$

Here, a_1, a_2 ……………a_n are the absorption coefficients of the materials in the hall whose surface areas exposed to sound are S_1, S_2……………….S_n respectively, measured in m^2.

The average value of absorption coefficient $\bar{a}$ is given by,

$$\bar{a}=\frac{a_1S_1+a_2S_2+\cdots.a_nS_n}{S_1+S_2+\cdots S_n}=\sum\frac{aS}{S}$$

$$\text{or} \sum aS=\bar{a}S$$

From the Sabine's formula for reverberation time, the reverberation time, T is,

i. Directly proportional to the volume of the auditorium.
ii. Inversely proportional to the areas of sound absorbing surfaces such as ceiling, walls, floor and other materials present inside the hall and
iii. Inversely proportional to the total absorption.

3.4 Basic requirements of acoustically good hall

The basic requirements of an acoustically good hall are,

1) **The volume of the auditorium** is decided by the type of programme to be conducted there and also the number of seats to be accommodated. A musical hall requires a large volume where as a lecture hall requires a smaller volume. In deciding the volume of the hall, its height plays an imp role than its length and breadth. The ratio between the ceiling height and breadth should be 2:3. In deciding the volume of the hall, the following guidelines may be followed.
   i) 3.74-4.2 m^3 per seat in cinema theatres.
   ii) 2.8-3.7m^3 per seat in lecture halls.
   iii) 4.2-5.6m^3 per seat in musical halls.
2) **The shape of the wall and ceiling** should be so as to provide uniform distribution of sound throughout the hall. The design of a hall requires smooth decay and growth of sound. To insure these factors, the hall should have scattering objects, walls should have irregular surface and walls must be fixed with absorptive materials.
3) **The reverberation of sound** in an auditorium is mainly due to multiple reflections at various surfaces inside. The reverberation should be optimum i.e., neither too large nor too small. The reverberation time should be 1-2 seconds for music and 0.5-1 sec for speech. To control the reverberation, the sound absorbing materials are to be chosen carefully.
4) **The sound heard** must be sufficiently loud in every part of the hall and no echoes should be present.
5) **The total quality of the speech and music** must be unchanged i.e., the relative intensities of the several components of a complex sound must be maintained.

6) For the sake of **clarity,** the successive syllables spoken must be clear and distinct i.e., there must be no confusion due to overlapping of syllables.

7) There should be no **concentration of sound** in any part of the hall.

8) The boundaries should be sufficiently **sound proof** to avoid noise from outside.

9) There should be no **echelon effect**.

10) There should be no **resonance** within the building.

11) The hall must be full of **audience**.

3.5 Absorption Co-efficient:

- The co-efficient of absorption of a material is defined as the ratio of the sound energy absorbed by the surface to that of the total sound energy incident on the surface i.e., Absorption co-efficient, a=$\frac{sound\ energy\ absorbed\ by\ the\ surface}{total\ sound\ energy\ incident\ on\ the\ surface}$
- An open window is considered as an ideal absorber of sound. The unit of sound absorption is open window unit or Sabine. A $1m^2$ Sabine is equal to the amount of sound energy that is absorbed or passed through one square meter area of open window.
- Absorption co-efficient of a surface is also defined as the reciprocal of its area which absorbs the same sound energy, as absorbed by unit area of open window.

3.5.1 Determination of Absorption Co-efficient

Method-1

The first method is based on the determination of standard times of reverberation in the room, without and with the sample of the material inside the room.

If T_1 is the reverberation time without the sample inside the room, then applying Sabine's formula,

$$\frac{1}{T_1} = \frac{A}{0.165V} = \frac{\sum aS}{0.165V}$$

Time T_2 is measured with the sample inside the room.

$$\frac{1}{T_2} = \frac{\sum aS + a_1 S_1}{0.165V}$$

Where, a_1 is the absorption coefficient of the area S_1

$$\text{Now, } \left(\frac{1}{T_2} - \frac{1}{T_1}\right) = \frac{a_1 S_1}{0.165V}$$

$$a_1 = \left(\frac{0.165V}{S_1}\right)\left(\frac{1}{T_2} - \frac{1}{T_1}\right) \qquad \rightarrow (1)$$

Hence, knowing the terms on the right hand side of the equation (1), the absorption coefficient a_1 of the given material can be calculated.

Method-2

This method consists of finding times of decay of the steady energy density, to the bear audibility for two sources of power outputs P_1 and P_2 respectively. From the equation of the decay of energy density,

$$E = E_m e^{-\propto t}$$

$$\text{Where, } E_m = \frac{4P}{\upsilon A} \text{ and } \propto = \frac{\upsilon A}{4V}$$

υ - Velocity of sound

A - $\sum$ ads, the total absorption of all the surfaces on which sound falls.

E – Energy density in the room, t seconds after the source is cut off.

Let t_1 and t_2 be the respective times of decay of energy density to the base audible limit E_0 for sources of power outputs P_1 and P_2 respectively. Then,

$$E_0 = \frac{4P_1}{\upsilon A} e^{-\alpha t_1} \text{ and } E_0 = \frac{4P_2}{\upsilon A} e^{-\alpha t_2}$$

Dividing the two equations,

$$\frac{P_1}{P_2} = e^{\alpha(t_1 - t_2)}$$

$$log_e\left(\frac{P_1}{P_2}\right) = \alpha(t_1 - t_2)$$

$$log_e\left(\frac{P_1}{P_2}\right) = \frac{\upsilon A}{4v}(t_1 - t_2)$$

$$\Rightarrow A = \frac{4V log_e\left(\frac{P_1}{P_2}\right)}{\upsilon(t_1 - t_2)}$$

$$\text{(or) } aS = \frac{4V log_e\left(\frac{P_1}{P_2}\right)}{\upsilon(t_1 - t_2)}$$

Where, ‘a’ is the average coefficient of absorption,

$$\boxed{a = \frac{4V log_e\left(\frac{P_1}{P_2}\right)}{\upsilon S(t_1 - t_2)}}$$

‘a’ can be calculated knowing the quantities on the right hand side of the equation.

3.6 Factors affecting the Architectural acoustics and their remedies

Following factors affect the architectural acoustics.

1) Reverberation

- In a hall, when reverberation is large, there is overlapping of successive sounds which results in loss of clarity in hearing. On the other hand, if the reverberation is very small, the loudness is inadequate. Thus, the reverberation time for a hall should neither to be too large nor too small. *The preferred value of reverberation time is called the Optimum reverberation time.*
- Experimentally it is observed that the time of reverberation depends upon the size of the hall, loudness of sound and on the kind of the music for which the hall is used.
- For a frequency of 512 Hz, the best time of reverberation lies between 1 and 1.5 sec for small halls and for large ones, it is up to 2-3 seconds.

Remedy: The reverberation can be controlled by the following factors.

i. By providing windows and ventilators which can be opened and closed to make the value of time of reverberation, optimum
ii. Decorating the walls by pictures and maps.
iii. Using heavy curtains with folds.
iv. By lining the walls with absorbent materials such as felt, fiber board etc.
v. Having full capacity of audience.
vi. By covering the floor with carpets.
vii. By providing acoustic tiles.

2) Loudness

With large absorption, the time of reverberation will be smaller and the intensity of sound may go below the level of hearing. Sufficient loudness at every point in the hall is an important factor for satisfactory hearing.

Remedy: The loudness may be increased by,

i. Using large sounding boards behind the speakers and facing the audience.
ii. Low ceilings are of great help to reflect the sound energy towards the audience.

iii. Providing additional sound energy with the help of equipments like loud speakers. For uniform distribution of intensity throughout the hall, the loudspeakers should be polished carefully.

3) Focusing

If there are focusing surfaces such as concave, spherical, cylindrical or parabolic ones on the walls or ceiling of the hall, they produce concentration of sound in particular regions, while in some other parts, no sound reaches at all. In this way, there will be regions of silence.

Remedy: For uniform distribution of sound energy in the hall,

i. There should be no curved surfaces. If such surfaces are present, they should be covered with absorbent material.
ii. Ceiling should be low.
iii. A paraboloidal reflected surface, with the speaker at the focus is also helpful in sending a uniform reflected beam of sound in the hall.

4) Echoes

An echo is heard when direct sound waves coming from the source, and it's reflected wave, reach the listener with a time internal of about 1/7 second. The reflected sound arriving earlier helps in raising the loudness while those arriving later produce echoes and confusion.

Remedy: Echoes may be avoided by covering the long distant walls and high ceiling with absorbent material.

5) Echelon effect

A musical note produced due to the combination of echoes, having regular phase difference is known as Echelon effect. The reflected sound waves from regularly spaced reflecting surfaces such as equally spaced stair cases or a set of railings produce musical note due to the regular succession of echoes of the original sound to the listener. This makes the original sound confused or unintelligible.

Remedy: Echelon effect can be avoided by forming the staircases with unusual spacing between them and covering them with sound absorbing materials like carpet.

6) Resonance

Sometimes, window panes loosely fitted wooden portions, wall separators and hollows, start vibrating by absorbing the sound produced in the hall. These may create sound. Certain tones of the original music and the created sound combine to produce interference such that the original sound gets disturbed.

Remedy: Resonance can be suppressed by hanging a large number of curtains in the hall.

7) Noise

Generally, there are three types of noise. They are (a) Air-borne noise (b) Structure borne noise (c) Inside noise.

(a) Air-borne noise: The noise that enters the hall from outside through open windows, doors and ventilators is known as air-borne noise.

Remedy:

i. By using heavy glass doors, windows or ventilators.
ii. By using double wall-doors and windows with insulating material in between them.
iii. Forming double wall construction.
iv. By fixing doors and windows at proper places.
v. Air conditioning the hall and sealing the openings perfectly.

(b) Structure-borne noise: The noise that reaches through the Structures of buildings is known as Structural noise. The activity around the building may cause a structural vibration of the building. Ex: footsteps, operating machinery, street traffic etc;

Remedy:

i. By using double walls with air space in between them.
ii. By using anti-vibration mounts.
iii. By properly insulating the equipments such as refrigerators, lifts, fans etc.,
iv. By using carpets on the floor.

(c) Inside noise: The noise produced inside big halls or offices due to equipment such as air conditioners, type writers and fans is called inside noise. This noise may be minimized as follows.

Remedy:

i. Placing the machinery on sound absorbent pads.
ii. Using noise-free air conditioners.
iii. Covering the floor with carpets, walls, ceilings with sound absorbing materials.

Chapter-4 ULTRASONICS

4.1 Introduction:

Ultrasonic has important uses in recent years. The audible frequency range of a healthy human ear is 20 Hz to 20,000 Hz (20 kHz). The sound waves which have a frequency less than 20 Hz are called infrasonic. The sound waves which have frequency more than 20 kHz are called ultrasonic. The frequency of ultrasonic may go up to megahertz. The properties are similar to that of sound waves. Ultrasonic has a large number of applications in engineering and medical fields.

4.2. Generation of ultrasonic waves:

Ultrasonic sound cannot be produced using a loudspeaker. It is because the diaphragm of the loudspeaker cannot vibrate with such high frequency. The following methods can be used to produce ultrasonic waves.

4.2.1. Magnetostriction method:

Principle of magnetostriction:

When a magnetic field is applied along the length of the ferromagnetic rod (such as iron or nickel), a small elongation occurs in its length. This phenomenon is called magnetostriction.

The increase in length depends on the intensity of applied magnetic field and nature of ferromagnetic material used. When a ferromagnetic rod is placed in a solenoid carrying alternating current, the length of the rod increases and contracts at a frequency twice that of the frequency of alternating current. The amplitude of these vibrations is small. When resonance occurs between the natural frequency of the rod and alternating current, the amplitude of the vibration increases. The natural frequency of the rod is given by;

$$f = \frac{1}{2l}\sqrt{\frac{\gamma}{\rho}} \qquad \text{............. (1)}$$

Where, γ is Young's modulus, ρ is the density of the material of the rod and l is length of the rod.

Construction:

An oscillator designed with NPN transistor is used to generate alternating current. The experimental arrangement is shown in figure 4.1. The LC circuit is connected to the collector of the transistor, the coil L_S is connected to the base of the transistor and the emitter is grounded. When the switch 'S' is on, the collector

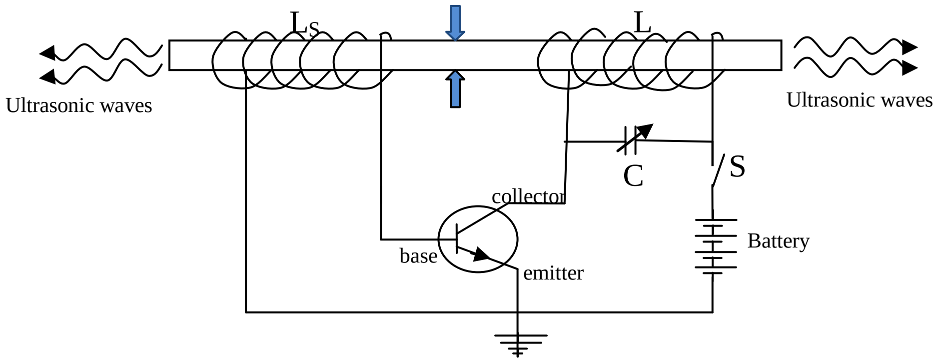


Figure 4.1. Magnetostriction oscillator

current starts increasing and oscillations start in LC circuit due to mutual inductance between L and L_S. The frequency of oscillations is given by;

$$f = \frac{1}{2\pi\sqrt{LC}} \qquad \text{.............} (2)$$

By varying C, the frequency can be changed and can be made equal to the natural frequency of the ferromagnetic rod. Under this resonance condition, ultrasonic waves are produced by the rod. By changing the length of the rod and the capacitance of the capacitor C, various high frequency oscillations can be obtained.

Merits:

1. Magnetostrictive material is easily available and low cost.
2. The oscillator circuit is easy to construct.
3. Required high frequency can be generated.

Demerits:

1. Frequencies beyond 300kHz cannot be produced.
2. As elastic constant changes with magnetization frequency of oscillations also change
3. Eddy current losses may occur due to a single rod of ferromagnetic material.

4.2.2. Piezoelectric method:

Principle of piezoelectric effect:

When one pair of opposite faces of piezoelectric material (such as quartz, Rochelle salt, Ammonium Dihydrogen Phosphate ADP) undergoes pressure electric charges are developed on the other pair of opposite faces. Instead of applying pressure on opposite faces, if tension is created in the opposite direction to the pressure (to pull the surfaces away) the charges get reversed on to the opposite faces. The converse of this effect, that is "if an alternating voltage is applied to one set of opposite faces, the length along the other set of opposite faces either increase or decrease depending on the direction of applied potential. The frequency of oscillations of the crystal depends on number of AC cycles. The natural frequency of the crystal is given by,

$$f = \frac{1}{2l}\sqrt{\frac{\gamma}{\rho}} = \frac{\upsilon}{2l} \qquad \text{.............. (3)}$$

Where, γ = Elastic modulus of material of the crystal

ρ = Density of material of the crystal

l = Length of material of the crystal between oscillating surfaces

υ = Velocity of longitudinal waves

Construction:

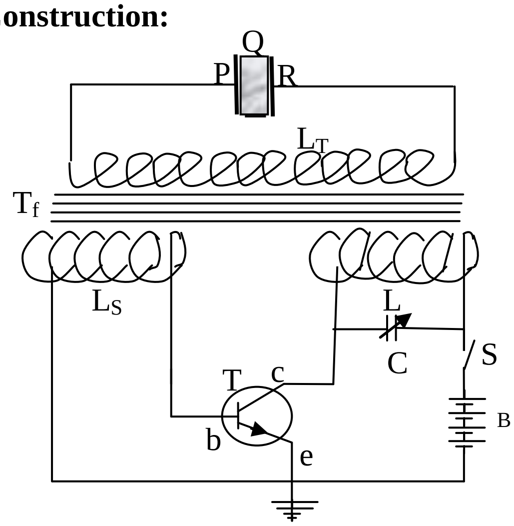


Figure 4.2. Piezoelectric oscillator

Q = Quartz crystal
P, R = Metal connectors
L, L_S, L_T = Coupled induction coils
C = Variable capacitor
S = Switch
T = Transistor
T_f = Transformer
B = Battery
c = Collector
b = Base
e Emitter

The above circuit is used to produce ultrasonic waves by the piezoelectric effect. A piezoelectric quartz crystal 'Q' is placed between two metal connectors, P and R which are connected to coupled induction coil L_T.

Working:

When the supply is switched on, the collector current increases and LC circuit connected to collector produces oscillations. The changes of current in L are fed back to base-emitter circuit by mutual inductance between L and L_S. The frequency of oscillations is given by

$$f = \frac{1}{2\pi\sqrt{LC}}$$

The frequency of oscillations can be varied by varying the capacity of the capacitor C. Due to mutual in the transformer T_f, an alternating emf is produced in the coil L_T. This alternating emf fed to electrodes P and R. The quartz crystal Q kept between electrodes P and R, experience oscillating electric force. Due to the inverse piezoelectric effect, the other pair of opposite faces in the crystal Q will oscillate with high frequency and thus ultrasonic waves are produced in quartz crystal. The capacity of the capacitor can be varied and made equal to the natural frequency of the quartz crystal so that resonance occurs and high frequency is generated. The frequency of the oscillation is given by;

$$f = \frac{1}{2l}\sqrt{\frac{\gamma}{\rho}} = \frac{\upsilon}{2l}$$

Merits:

1. Frequencies up to 500 MHz can be produced.
2. The oscillator circuit is easy to construct.
3. Required high frequency range can be generated.

Demerits:

1. Piezoelectric crystals are expensive.
2. Cutting piezoelectric crystal is difficult.

4.3. Properties of ultrasonic waves:

1. The human ear cannot hear ultrasound.
2. These are acoustic waves with a frequency greater than 20 kHz.
3. The wavelength of ultrasonic waves is small.
4. The diffraction of the wave is very less.
5. They can travel long distances. Hence they are used in SONAR.

6. These waves can also produce acoustic grating.
7. As frequency is very high, these waves are more energetic since E = hν.
8. When these waves are absorbed any medium through which they pass, it gets heated.

4.4. Applications of ultrasonic waves:

4.4.1. Engineering applications:

SONAR:

SONAR stands for "**SO**und **N**avigation **A**nd **R**anging". Sonar is used to detect submerged objects underwater such as submarines, sinked boats inside the sea or depth of the sea. It is similar to RADAR in the air.

Let the depth of the sea or any forgin object in the sea be x . Let υ be the velocity of ultrasonic wave, t be the time interval between emitted ultrasonic signal and received echo signal after reflection from the base then,

$$x = \frac{\upsilon \times t}{2}$$

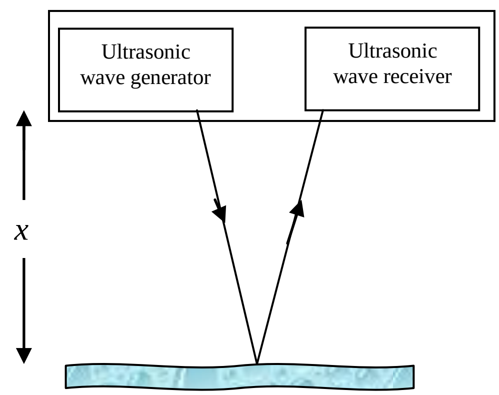


Figure 4.3. Measuring depth of sea using SONAR

Drilling, cutting and soldering of metals:

Ultrasonic waves can be used for drilling, cutting and soldering in metals at room temperature.

Cleaning:

Ultrasonic waves can also be used as a cleaning agent to clean clothes, utensils, removing dust and soot from chemney.

Chemical reactions:

These waves act as a catalyst to start and increase the rate of reaction.

Signaling systems:

Ultrasonic waves are more energetic and have less diffracting property, so they travel long distances in air and water even sea water. Hence, they can be used in signaling systems in air and sea water.

Non-destructive testing (NDT):

Widely used testing method is non-destructive testing. High frequency sound waves are used to test dimensional measurements, meterial characterization, flow detection and evaluation.

4.4.2. Medical applications:

Ultrasonic waves are used in various diagnostic and therapeutic applications.

Estimation of velocity of blood flow in veins and arteries:

The Doppler shift in higher ultrasound frequencies is used to estimate the velocity of blood flow in veins and arteries.

Ultrasonic scanning for heart, prostate enlargement detection and sonogram (echogram) of pregnant woman:

Low energy ultrasound from the transducer is used to study the movement of the heart, prostate enlargement in ultrasound scanning.

A sonogram (or echogram) of a pregnant woman can also be obtained using ultrasound scanning, which shows the fetal growth and study of the bodily organs in the womb.

Diagnostic use of ultrasound:

Ultrasonic waves are used in detectecting tumers or any other defects in the body.

Rheumatic and neuralgic pains:

When the ultrasonic waves are exposed on the part where the rheumatic or neuralgic pains occur, the massage action of the waves relieves the pain.

Sterilization:

Ultrasonic waves sterilize water and milk.

4.4.3. Other applications ultrasonic waves:

Ultrasonic wet-milling and grinding

Ultrasonics is an efficient means for the wet-milling and micro-grinding of particles. In particular for the manufacturing of superfine-size slurries, ultrasound has many advantages, when compared with common size reduction equipment, such as: colloid mills (e.g. ball mills, bead mills), disc mills or jet mills.

Ultrasonic cell extraction

Ultrasonics is used in the extraction of enzymes and proteins stored in cells and subcellular particles. The extraction of organic compounds contained within the body of plants and seeds by a solvent can be done using ultrasound.

Ultrasonic degassing of liquids

In this case the ultrasound removes small suspended gas-bubbles from the liquid and reduces the level of dissolved gas below the natural equilibrium level.

Ultrasonic leak detection of bottles and cans:

Ultrasound is being used in bottling and filling machines to check cans and bottles for leakage.

Ultrasonic wire, cable and strip cleaning:

Ultrasonic cleaning is an environmentally friendly. The cleaning of continuous materials, such as wire and cable, tape or tubes can be done with ultrasonic cleaning process. The ultrasonic power removes lubrication residues like oil or grease, soaps, stearates or dust.

UNIT -III

Chapter - 5. EQUILIBRIUM OF SYSTEM OF FORCES

5.1. Basic Definitions

Particle: Point size mass (m ~ 0) of material which has negligible dimensions in the study of motion & equilibrium.
Ex: Earth & Moon in case of whole universe, electron in case of atomic structure

System of particles: the system of particles which is constituted with two or more bodies are dealt together.

Rigid Body: A rigid body is one in which the distance between any two arbitrary points is always constant.

Space: it is geometric region containing bodies. Positions of the bodies are described by linear and angular measurements relative coordinate system.

(i) Point: Just an exact location in space has no space at all, doesn't occupy any space having coordinates (x, y, z) (r, ϴ) (r, l, ϴ)

(ii) Length: the measurements between two independent points measured in [L], cm, m. It is Scalar Quantity.

(iii) Mass: It is quantity of matter. Matter refers to substance contained in the physical bodies. It is measured in [M]. g. Kg. It is Scalar quantity.

(iv) Time: Measure of sequence of events. It is time related concept, before and after simultaneous occurrence. It is basic quantity in dynamics. It is not evolved with statics. It is measured in [T]. Units Seconds. It is Scalar quantity.

Other physical quantities:

Velocity: Length moved in unit time. It is vector quantity

$$\text{Velocity, } V = \frac{L}{T} = [LT^{-1}] \qquad \text{m/s or cm/s}$$

Acceleration: Change in velocity per unit time

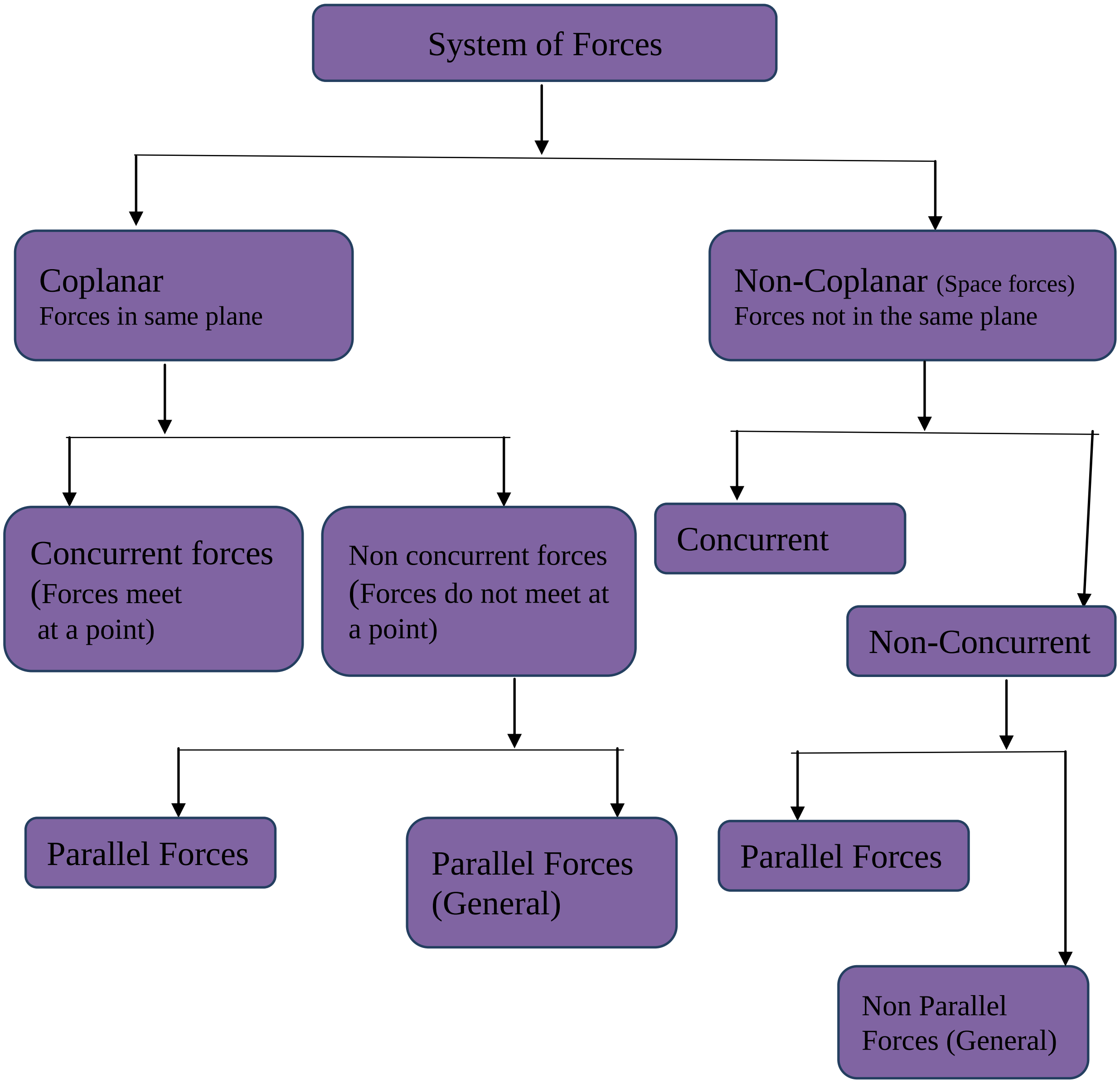
$$\text{Acceleration} = \frac{V}{T} = [LT^{-2}] \qquad m/s^2 \text{ or } cm/s^2$$

Scalar has only magnitude but vector both magnitude and direction.

Force: It is action that tends to change the state of rest of a body. It is dynamic. It is characterized by (i) Magnitude (ii) Direction (iii) point of Application. It is a vector quantity $[MLT^{-2}]$

Newtonian force $F= m * a$ $Kg\ m\ s^{-2}$ or Newton or Dyne

5.2. System of Forces:



5.3. Fundamental and Derived Units

Fundaments units are seven (7)

M – Mass;	Kg, g	K – Temperature;	Kelvin
L – Length;	m, cm	A – Electric Current;	Ampere
T – Time;	Seconds	Cd - Luminous Intensity;	Candella
Mol – Amount of Substance;	mole		

Derived units: The units which are derived from fundamental units are derived units.

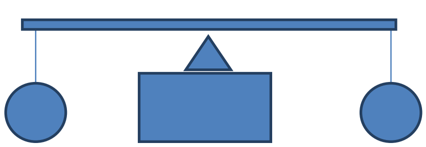
Momentum: mass x Velocity	Kg ms^{-2}	$[MLT^{-2}]$
Work: force x distance	Nm	$[ML^{2} T^{-2}]$
Pressure: Force/ Area	N/m^{2}	$[ML^{-1} T^{-1}]$
Frequency : Number of vibrations	Hz	$[T^{-1}]$
Angular acceleration:	rad/ S^{2}	$[T^{-2}]$
Angular Momentum:	Kg m^{2}/ second	$[ML^{2} T^{-1}]$

5.4. Concurrent Forces in a Plane

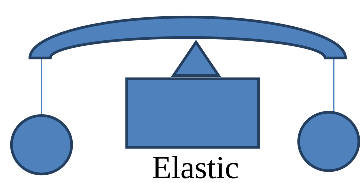
Rigid Body: The body in which definite quantity of matter in the parts of is fixed relative to one another is called rigid body.

Or

Some of the physical bodies deform slightly i.e. SAG is very less (SAG= state of sinking or bending) when external load is applied. If SAG is negligible, it is called as a rigid body. If the SAG is more, it is called elastic body. Elastic coefficient is proportional to SAG.



Rigid



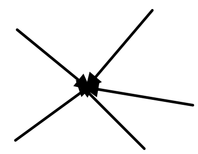
Elastic

Force: Action that tends to change the state of rest or motion of body to which it is applied is called force. The force produces linear displacement or angular displacement or both, essentially results in change of state of rest or motion of the body.

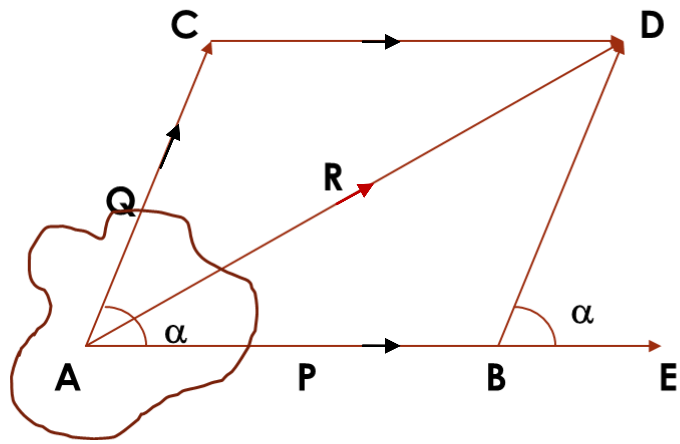
Weight is also a kind of force called gravitational force, Hydrostatic pressure, gas pressure, wind pressure are different forms of forces.

5.5. Concurrent forces in a plane

Various forces meeting at a point is called concurrent forces in plane.



Composition of two or more forces in a plane



Force is vector quantity

$\vec{P} + \vec{Q} = \vec{R}$ (Resultant)

$\vec{AB}$, $\vec{AC}$ are components $\vec{P}$, $\vec{Q}$ are constructing parallelogram ABCD as shown in Figure.

And let α be the angle between both the components P and Q.

BE= Q cosα, DE= Q sinα

$\vec{AE} = \vec{P} + \vec{Q}$ Cosα

$\vec{R}^2 = AE^2 + ED^2$

$R^2 = (P + Q \cos\alpha)^2 + (Q \sin\alpha)^2$

$= P^2 + Q^2 \cos^2\alpha + 2PQ \cos\alpha + Q^2 \sin^2 \alpha$

$= P^2 + Q^2 (\cos^2\alpha + \sin^2 \alpha) + 2PQ \cos\alpha$

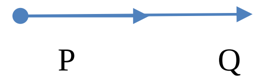
$R^2 = P^2 + Q^2 + 2PQ \cos\alpha$

$|R| = \sqrt{P2 \ + \ Q2 \ + \ 2PQ \cos\alpha}$

Special cases:

When $\alpha = 0°$; $R = \sqrt{P2 \ + \ Q2 \ + \ 2PQ}$

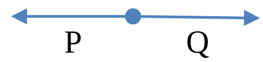
$R = P + Q$



When $\alpha = 180°$; $R = \sqrt{P2 \ + \ Q2 \ + \ 2PQ}\,(-1)$

$R = \sqrt{P2 \ + \ Q2 - \ 2PQ}$

$R = P - Q$



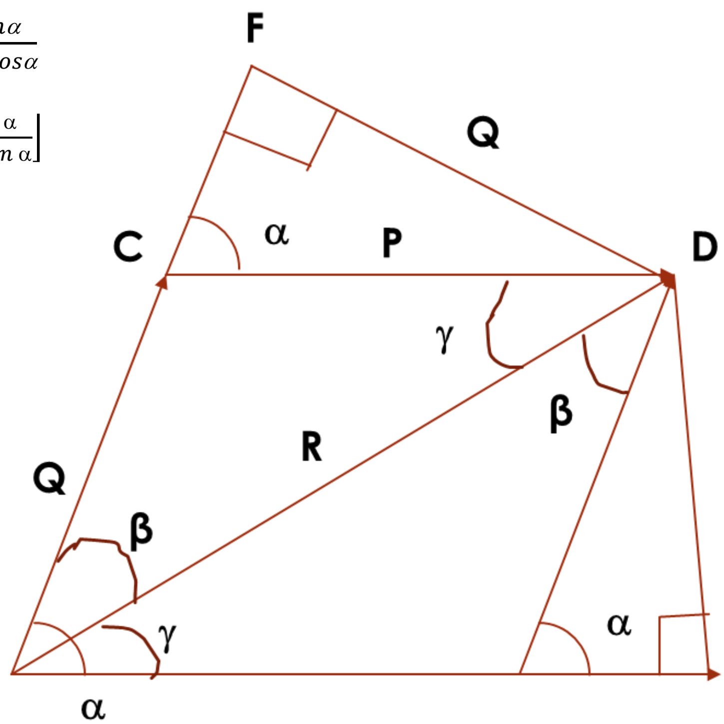
When $\alpha = 90°$; $R = \sqrt{P2 \ + \ Q2 \ + \ 2PQ \cos 90}$

$R = \sqrt{P2 \ + \ Q2}$

$|R|$ is magnitude of resultant force.

For Direction $\quad \text{Tan}\ \gamma = \frac{DE}{AE} = \frac{Q\sin\alpha}{P+Q\cos\alpha}$

$\gamma = \text{Tan-1} \left[\frac{Q\ Sin\ \alpha}{P+Q\ Sin\ \alpha}\right]$



$\text{Sin}\ \gamma = \frac{DE}{AD} = \frac{Q\sin\alpha}{R}$

In triangle CFD,

$\angle C = \angle CAB = \alpha$; CD= P

FD = P sinα

In triangle CFD,

$\text{Sin}\ \beta = \frac{DF}{AD} = \frac{P\sin\beta}{R}$

$$\frac{\sin\alpha}{R} = \frac{\sin\gamma}{Q} = \frac{\sin\beta}{P}$$

$$\frac{\sin\beta}{\sin\gamma} = \frac{P}{Q}$$

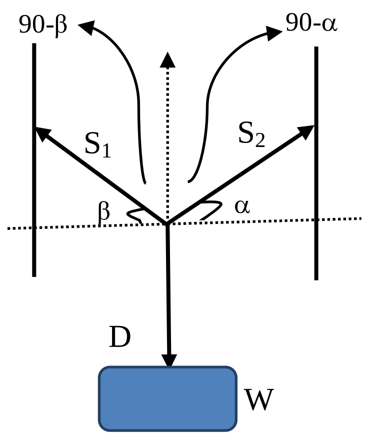
5.5.1. Case study

Force along BC = $S_2 \cos(90-\beta)$
$= S_2 \sin\beta$

Force along BC = $S_1 \sin\alpha$

The total force along BC = $S_1 \sin\alpha + S_2 \sin\beta$

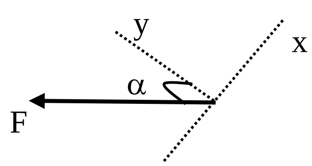
Resultant force, action & reaction are equal and opposite to keep the ring constant in equilibrium position.



5.6. Resolving Force into Rectangular components

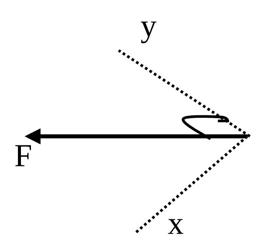
(1) $F_x = - F \sin \alpha$

$F_y = F \cos \alpha$

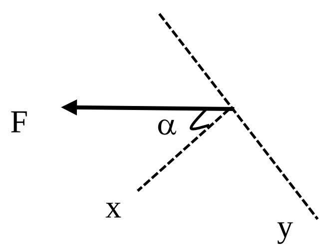


(2) $F_x = F \sin \alpha$

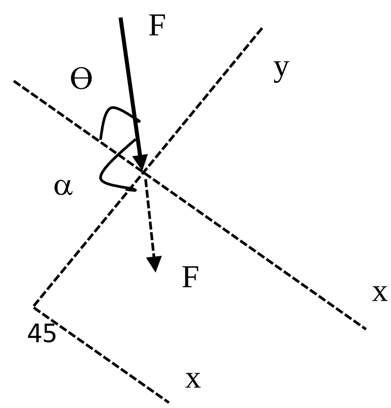
$F_y = - F \cos \alpha$



(3) $F_x = F \cos\alpha$
$F_y = - F \cos\alpha$



(4) $F_y = F \cos(180 - \alpha)$
$= - F \cos\alpha$
$\Theta = 90 - (180 - \alpha)$
$= - 90 + \alpha = - (90 - \alpha)$



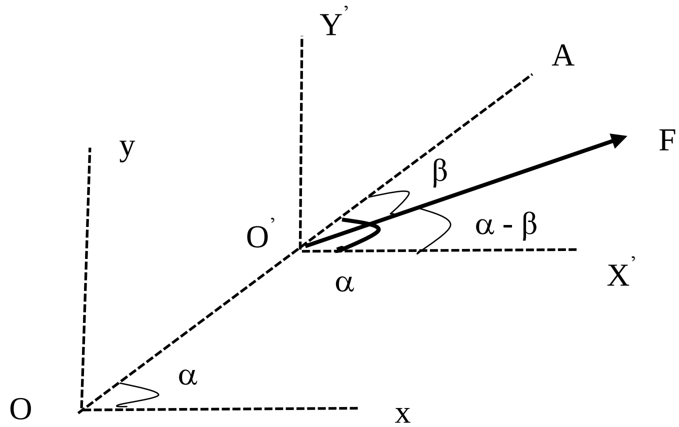
$F_x = - F \cos \Theta$

$= - F \cos [(- (90- \alpha))]$

$= F \cos (90- \alpha) = F \sin\alpha$

(5) $F_x = F \cos(\alpha-\beta)$

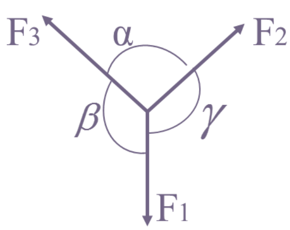
$F = F \sin(\alpha-\beta)$

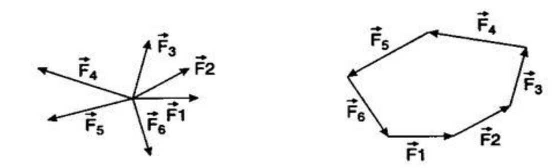


5.7. Condition for Equilibrium of concurrent forces in plane

Condition for equilibrium for concurrent forces in plane

If a body acted by several concurrent forces in equilibrium, the free vectors geometrically added must form a closed polygon.





(a) Concurrent coplanar forces (b) Polygon of force vectors

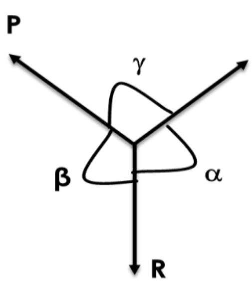
By triangle law of forces

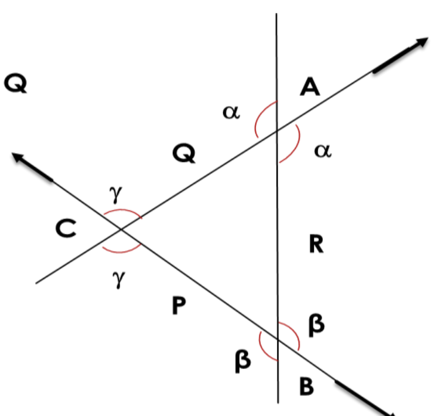
$F_1+F_2+F_3 =0$ or $F_1+F_2 = -F_3$

By polygon law of forces

$F_1+F_2+F_3+F_4+F_5+F_6 = 0$

5.8. Lami's Theorem





If three concurrent forces are acting on a body and kept it in equilibrium, then the ratio of each force and sine of angle between them, other two forces is always constant.

$$\frac{P}{sin\alpha} = \frac{Q}{sin\beta} = \frac{R}{sin\gamma} = \mathrm{K}$$

In Triangle ABC,

$\angle A = \Pi - \alpha$; $\angle B = \Pi - \beta$; $\angle C = \Pi - \gamma$

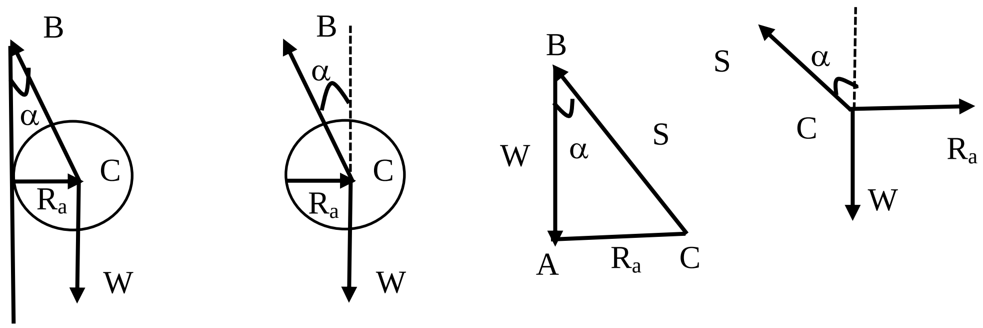
From Sine rule of Triangle,

$\frac{P}{sinA} = \frac{Q}{sinB} = \frac{R}{sinC}$

$= \frac{P}{sin\pi-\alpha} = \frac{Q}{sin\pi-\beta} = \frac{R}{sin\pi-\gamma}$

$= \frac{P}{sin\alpha} = \frac{Q}{sin\beta} = \frac{R}{sin\gamma}$ (since sin (180-Θ) = sinΘ)

5.8.1. Application



(a) **(b)** **(c)** **(d)**

In the triangle of figure (c), from trigonometric solutions R_a and S can be computed as

Tan $\alpha = R_a$ / W ; R_a = W Tanβ

Cos α = W/S ; S = W cosα

From Lami's Theorem, in Triangle ABE

$\frac{R_a}{sinB} = \frac{W}{sinC} = \frac{S}{sinD}$

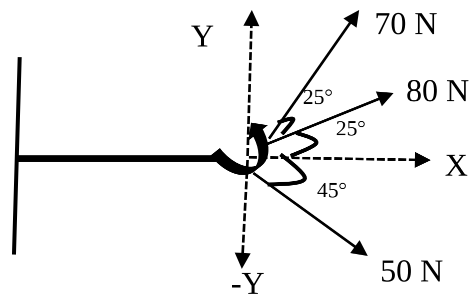
$\frac{R_a}{sin\alpha} = \frac{W}{\sin{(90-\alpha)}} = \frac{S}{sin90}$

$\frac{R_a}{sin\alpha} = \frac{W}{\cos\alpha} = S$

From this, $\frac{sin\alpha}{cos\alpha}$ = Tanα = $\frac{R_a}{W}$ and secα = $\frac{S}{W}$

Problem

Determine the resultant of three forces



Forces 70 N, 80 N, 50 N

x- components

70 cos 50 = 45.00
80 cos 25 = 72.50
50 cos 45 = 35.36

Therefore ∑ x = 152.86

y- components

70 sin50 = 53.62
80 sin25 = 33.81
-50sin45 = -35.36
Therefore ∑ y = 52.07

R = $\sqrt{(152.86^2) + 52.07^2}$ = 161. 41 N

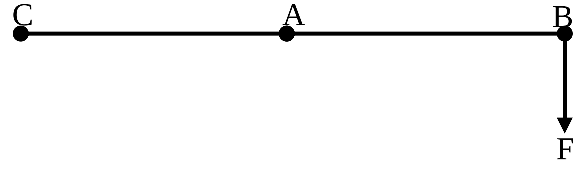
α = Tan^{-1} [$\frac{\sum x}{\sum y}$] = Tan^{-1} [$\frac{52.07}{152.86}$] = 18.81° from x-axis

5.9. Moment of Force:

Moment of force (M) about a point is a measure of rotational effect at that point. Moment of force about any point is defined as the product of magnitude of force (F) and perpendicular distance of line of force to that point.

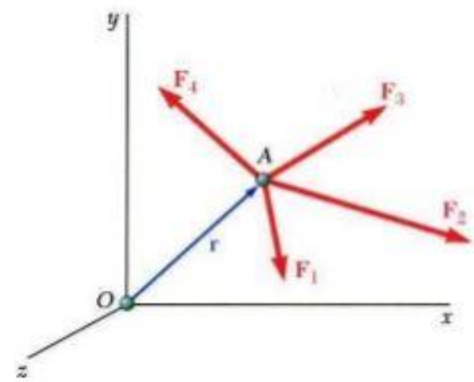
Moment of force about point A= F x (AB)

Moment of force about point C = F x (BC)



5.10. Varignon's Theorem

The Algebraic sum of moments of a system of coplanar forces about any point is equal to moment of their resultant forces about the same centre.



Then According to

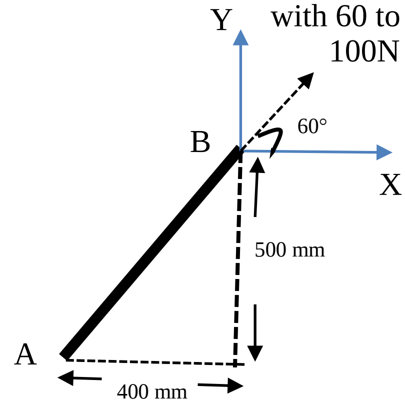
Varigon's Theorem

Resultant force = r x (F_1 +F_2 +F_3 +F_4)

Problem

A force of 100N is applied at one end of a rod with 60 to horizontal as shown in figure. Find the moment of Force about A.

(Consider Clockwise direction as Positive)



M_A = [100 cos60° x 500] – [100 sin60° x 400]

= 25000 – 34,641

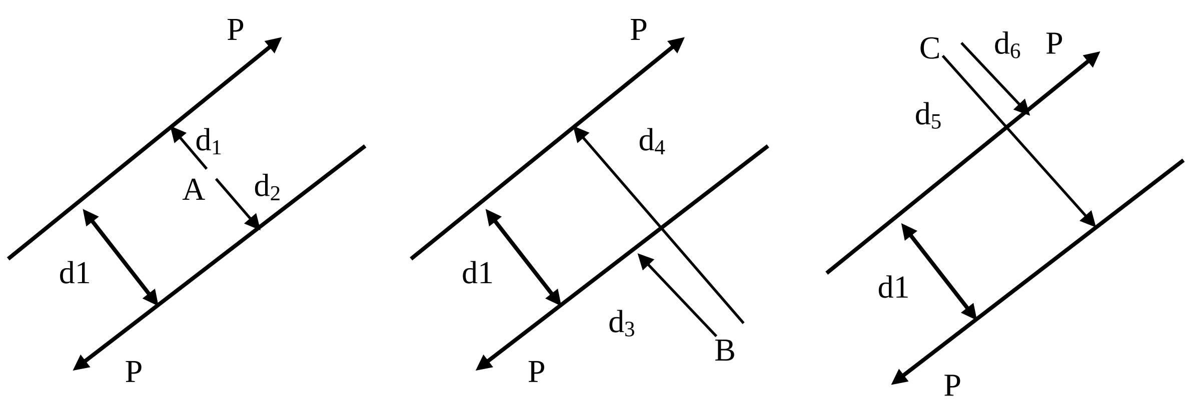
= -9,641 N mm

⇒ 9,641 N-mm is in anti-clockwise Direction

5.11. Couple:

Two parallel forces equal in magnitude and opposite in direction forms a couple.

The rotational effect is considered as positive in clockwise direction and negative in anti-clock wise direction.



(a) (b) (c)

$M_A = Pd_1 + Pd_2$
$= P(d_1 + d_2)$
$= Pd$

$M_B = Pd_3 - Pd_4$
$= P(d_3 - d_4)$
$= Pd$

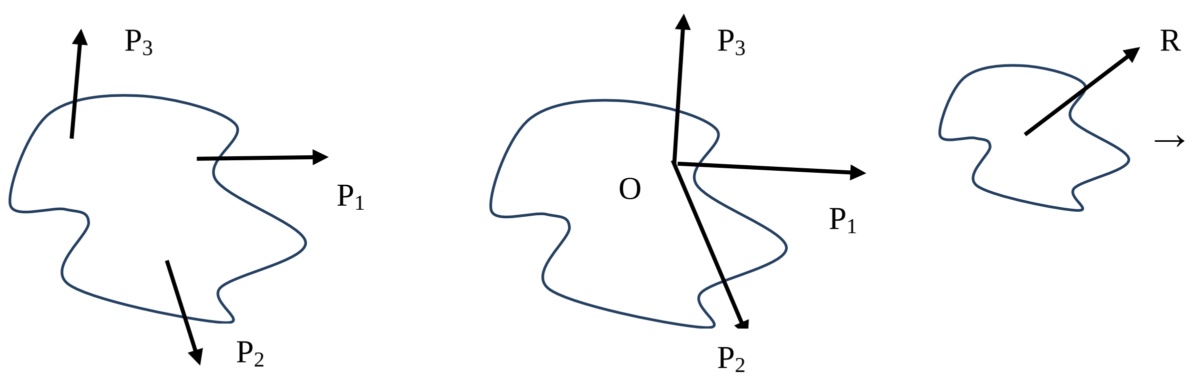
$M_c = Pd_5 - Pd_6$
$= P(d_5 - d_6)$
$= Pd$

The moment of couple about any point is same

5.11.1. Characteristics of Couple

1. Consists of pair of equal and opposite forces separated by a definite distance.
2. Translatory effect of couple on the body is zero.
3. Rotational effect (moment) of a couple about any point is constant and it is equal to product of magnitude of force and perpendicular distances between the forces.
4. The moment of couple is same, the effect is unchanged
    a. When couple rotated through any angle
    b. When couple shifted to any other position
    c. When is couple is replaced by any other pair of forces

5.12. Resultant of System of Forces



$R_x = P_{1x} + P_{2x} + P_{3x}$, $R_y = P_{1y} + P_{2y} + P_{3y}$

$R = \sqrt{(\sum x_i)^2 + (\sum y_i)^2}$

$\tan\alpha = \frac{\sum y}{\sum x}$; $d = \frac{\sum M_o}{R}$

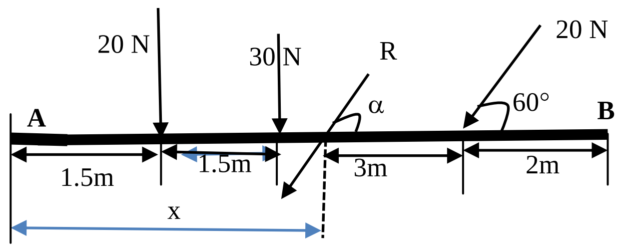
$\sum x$ = algebraic sum of all components in x- direction

$\sum y$ = algebraic sum of all components in y- direction

α = Inclination of R with x-direction d = Perpendicular distance from R to O

Problem

A system of load is acting on a beam is shown in figure. Calculate resultant force R of the loads.



Solution

$\sum x = 20 \cos 60° = 10$ N

$\sum y = 20 + 30 + 20 \sin 60° = 67.32$ N

R = $\sqrt{(\sum x)^2 + (\sum y)^2} = \sqrt{10^2 + 67.32^2} = 68.06\ N$

Tanα = $\frac{\sum x}{\sum y} = \frac{67.32}{10} = 6.73 \quad \Rightarrow \alpha = 81.55°$

Moment about A = $\sum M_A$ = (20 x 1.5) + (30 x 3) + (20 sin60 x 6)

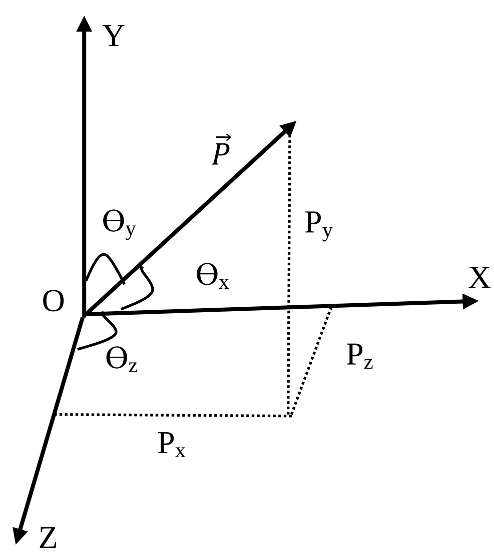
= 223. 92 N-m

Distance of resultant d = $\frac{\sum M_A}{R} = \frac{223.\ 92}{68.06} = 3.29$ m

x = $\frac{d}{sin\alpha} = \frac{3.29}{sin 81.55} = 3.326$ m

x intercept = $\frac{\sum M_A}{R_y} = \frac{\sum M_A}{\sum y}$ = 223.93/ 67.32 = 3.326 m

5.13. Resultant of Concurrent forces in space



Let x, y, z are co-ordinate axes

P is a line of force from origin

Θ_x is the angle made by P with X-axis

Θ_y is the angle made by P with Y-axis

Θ_z is the angle made by P with Z-axis

Resolving P into X, Y and Z components

$P_x = P \cos \Theta_x$

$P_y = P \cos \Theta_y$ (1)

$P_z = P \cos \Theta_z$

Let A_i, B_j be two points on the line of action of $\vec{P}$ with Coordinates (x_i , y_i , z_i) and (x_J , y_j , z_j) respectively.

From co-ordinate geometry; The distance between A_i , B_j is given by

$$L = \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2 + (z_j - z_i)^2}$$

From trigonometric relations

$$\cos \Theta_x = \frac{(x_j - x_i)}{L}$$

$$\cos \Theta_y = \frac{(y_j - y_i)}{L}$$

$$\cos \Theta_z = \frac{(x_j - x_i)}{L}$$

The resultant of several concurrent forces P_1, P_2 , P_3………. Can be determined using their component forces P_{1x}, P_{1y}, P_{1z}, P_{2x} P_{2y} P_{2z}, P_{3x} P_{3y} P_{3z} ………….. etc.

x component of resultant R, $R_x = \sum P_x = P_{1x} + P_{2x} + P_{3x} + \ldots\ldots\ldots\ldots$

y component of resultant R, $R_y = \sum P_y = P_{1y} + P_{2y} + P_{3y} + \ldots\ldots\ldots\ldots$

z component of resultant R, $R_z = \sum P_z = P_{1z} + P_{2z} + P_3 + \ldots\ldots\ldots\ldots$

And Resultant $R = \sqrt{R_x^2 + R_y^2 + R_z^2}$

$$R = \sqrt{(\sum P_x)^2 + (\sum P_y)^2 + (\sum P_z)^2}$$

$\Theta_x = \cos^{-1}\left(\frac{R_x}{R}\right)$; $\Theta_y = \cos^{-1}\left(\frac{R_y}{R}\right)$; $\Theta_z = \cos^{-1}\left(\frac{R_z}{R}\right)$

Problem:

Four forces of 10N, 20N, 25N and 40N are concurrent in space at origin and passing through points (2, 3, 5); (1,7,4) ; (4,-2, 4) and (-2, 3, 4) respectively. Determine the resultant of system of forces.

Solution:

P_A = 10N ; P_B = 20N ; P_C = 25 N ; P_D = 40 N

Let O = (0, 0, 0) A= (2, 3, 5); B= (1,7,4) ; C = (4,-2, 4) and D = (-2, 3, 4)

Length OA = $\sqrt{(3-0)^2+(2-0)^2+(5-0)^2}$ = 6.16

Similarly OB = 8.12 OC = 6.0 OD = 5.385

Cos Θ_{Ax} = $\frac{(A_x - O_x)}{OA}$ = $\frac{3-0}{6.16}$ = 0.487

Cos Θ_{Ay} = $\frac{(A_y - O_y)}{OA}$ = 0. 3245

Cos Θ_{Az} = $\frac{(A_z - O_z)}{OA}$ = 0. 0.811

P_{Ax} = P_A Cos Θ_{Ax} = 10 ∗ 0.4876 = 4. 87 N

P_{Ay} = P_A Cos Θ_{Ay} = 10 ∗ 0. 4245 = 3.245 N

P_{Az} = P_A Cos Θ_{Az} = 10 ∗ 0. 8112 = 8.11 N

Similarly

Cos Θ_{Bx} = 0.1231 Cos Θ_{By} = 0. 8616 Cos Θ_{Bz} = 0. 492

P_{Bx} = P_B Cos Θ_{Bx} = 20 ∗ 0. 1231 = 2. 462 N

P_{By} = 17. 232 N P_{Bz} = 9. 8848 N

Similarly

Cos Θ_{Cx} = 4/6 Cos Θ_{Cy} = -2/6 Cos Θ_{Cz} = 4/6

P_{Cx} = 25 ∗ (4/6) = 16. 6 N ; P_{Cy} = 8. 333 N P_{Cz} = 16. 667 N

Similarly

Cos Θ_{Dx} = -2/5.38 Cos Θ_{Dy} = 4/5.385 Cos Θ_{Dz} = -3/5.385

P_{Dx} = 40 ∗ (-2/5.385) = -14. 856 N ; P_{Dy} = 29. 71 N P_{Dz} = -22. 484 N

$R_x = \sum P_x$ = 4.867 + 2.462 + 16.667 -14.856 = 9.14 N

$R_y = \sum P_y = 3.245 + 17.232 – 8.333 + 29.712 = 41.856$ N

$R_z = \sum P_z = 8.112 + 9.848 + 16.667 – 22.284 = 12.343$ N

$R = \sqrt{\left(R_x^2 + R_y^2 + R_z^2\right)} = (9.12^2 + 4.18^2 + 12.243^2)^{1/2} = 4.85$ N

$R_{\Theta x} = \cos^{-1}\frac{R_x}{R} = \cos^{-1}\left(\frac{9.14}{44.58}\right) = 78.17°$

$R_{\Theta y} = \cos^{-1}\frac{R_y}{R} = \cos^{-1}\left(\frac{41.85}{44.58}\right) = 20.15°$

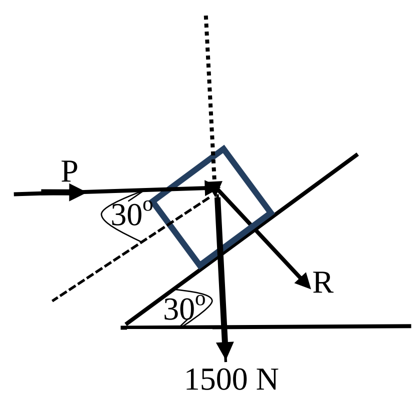
$R_{\Theta z} = \cos^{-1}\frac{R_z}{R} = \cos^{-1}\left(\frac{12.343}{44.58}\right) = 73.93°$

Problem

Determine Horizontal force P to be applied to a block of weight 1500 N to hold in a position on a smooth inclined plane AB of angle 30° with horizontal.

Solution:

R = Normal reaction
Angle of inclination = 30^o
Since the body is in equilibrium



$\sum Y = 0$

$R \cos 30^o – 1500 = 0$

$\Rightarrow R = 1732.06$ N

$\sum Z = 0$

$P - R \sin 30^o = 0 \Rightarrow P = R \sin 30^o = 866.06$N

2nd Method : Applying Lam's Theorem

$$\frac{R}{Sin 90} = \frac{P}{\text{Sin }(180-30)} = \frac{1500}{\text{Sin }(90+30)}$$

R = 1732. 06 N

P = 866. 03N