UNIT 8: MOMENT OF INERTIA

Structure

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8.1 INTRODUCTION

In the units 5 and 6, you have studied about angular momentum and its conservation, moment of inertia and its physical significance, radius of gyration, equations of angular motion and rotational kinetic energy. In this unit, we shall study some more important concepts of motion and laws of rotational motion. We know that, to find the moment of inertia of a body about a given axis, all that we have to do is to find the sum $\Sigma\ mr^2$ for all particles making up the body by integration or other means. The calculations to find the moment of inertia can be made shorter by the help of some important theorems. In this unit, we shall also study those theorems (theorems of parallel and perpendicular axes).

8.2 OBJECTIVES

After studying this unit, you should be able to-

- Solve problems based on equations of motion
- apply equations of motion
- understand laws of rotational motion
- apply theorems of parallel and perpendicular axes.

8.3 EQUATIONS OF MOTION

If an object is moving in a straight line under a constant acceleration, then relations among its velocity, displacement, time and acceleration can be represented by equations. These equations are called 'equations of motion'.

Let us consider that a body starts with an initial velocity 'u' and has a constant acceleration 'a'. Suppose it covers a distance's' in time't' and its velocity becomes 'v'. Then the relations among u, a, t, s and v can be represented by three equations.

First Equation: We know that linear acceleration $a = \frac{dv}{dt}$

or $dv = a\ dt$

Integrating both sides, we get-

$\int_u^v dv = \int_0^t a\ dt$

or $(v - u) = a\ (t - 0)$

or $v = u + at$ .....(1)

This is known as first equation of motion.

Second Equation: We know that linear velocity $v = \frac{ds}{dt}$

But v = u + at

Therefore, $u + at = \frac{ds}{dt}$

or $\frac{ds}{dt} = u + at$

or ds = (u + at) dt

Integrating both sides, we get-

$\int_0^s ds = \int_0^t (u + at)dt$

or $s = u(t - 0) + a(\frac{t^2}{2} - 0)$

or $s = ut + \frac{1}{2} a t^2$ .....(2)

This is called second equation of motion.

Third Equation: By first equation, v = u + at

Squaring both sides-

$v^2 = (u + at)^2$

or $v^2 = u^2 + a^2t^2 + 2uat$

$= u^2 + 2a(ut + \frac{1}{2} a t^2)$

or $v^2 = u^2 + 2as$ …..(3)

(using equation 2)

The above equation (3) is known as third equation of motion.

Example 1: A train starting from rest is accelerated by 0.5 m/sec^2 for 10 sec. Calculate its final velocity after 10 sec. Also calculate the distance travelled by train in 10 sec.

Solution: Here, u = 0, a = 0.5 m/sec^2, t = 10 sec

Using first equation of motion v = u + at

$v = 0 + 0.5 \times 10 = 5$ m/sec

Using second equation of motion $s = ut + \frac{1}{2} a t^2$

$s = 0\ (10) + \frac{1}{2} \times 0.5\ (10)^2$

$= \frac{1}{2} \times 0.5 \times 100 = 25$ m

Thus the velocity of train after 10 sec is 5 m/sec and the distance travelled is 25 m.

Example 2: A car is accelerated from 8 m/sec to 14 m/sec in 3 sec. What is the acceleration of car ?

Solution: Here, u = 8 m/sec, v = 14 m/sec, t = 3 sec

Using $v = u + at$

$14 = 8 + a \times 3$

$3a = 14 - 8$

$= 6$

$a = 2$ m/sec^2

Self Assessment Question (SAQ) 1: A car is moving with a constant speed of 30 Km/hr. Calculate the distance travelled by car in 1 hr.

Self Assessment Question (SAQ) 2: A particle is shot with constant speed 6×10^6 m/sec in an electric field which produces an acceleration of 1.26×10^{14} m/sec^2 directed opposite to the initial velocity. How far does the particle travel before coming to rest?

Self Assessment Question (SAQ) 3: The initial velocity of a particle is 'u' (at t = 0) and the acceleration is given by 'at^2'. Which of the following relations is valid?

(a) $v = u + at$ (b) $v = u + \frac{at^3}{3}$ (c) $v^2 = u + at^3$ (d) $v = u + \frac{at^3}{2}$

8.4 NEWTON'S LAWS OF ROTATIONAL MOTION

As we know, there are Newton's three laws of translational motion, similarly we have the following three laws of rotational motion-

First Law: Unless an external torque is applied to it, the state of rest or uniform rotational motion of a body about its fixed axis of rotation remains unaltered.

Second Law: The rate of change of angular momentum (or the rate of change of rotation) of a body about a fixed axis of rotation is directly proportional to the torque applied and takes place in the direction of the torque.

Third Law: When a torque is applied by one body on another, an equal and opposite torque is applied by the latter on the former about the same axis of rotation.

8.5 GENERAL THEOREMS ON MOMENT OF INERTIA

There are two important theorems on moment of inertia which, in some cases, facilitate the moment of inertia of a body to be determined about an axis, if its moment of inertia about some other axis be known. These theorems are theorem of parallel axes and theorem of perpendicular axes. Let us discuss these theorems.

8.5.1 Theorem of Parallel Axes

It states that the moment of inertia of a body about any axis is equal to its moment of inertia about a parallel axis through its centre of mass plus the product of the mass of the body and the square of the perpendicular distance between the two axes.

If 'I_{cm}' be the moment of inertia of a body about a parallel axis through its centre of mass, 'M' be the mass of the body and 'r' be the perpendicular distance between two axes, then moment of inertia of the body $I = I_{cm} + Mr^2$

This is the "theorem of parallel axes".

Proof: Let us consider a plane lamina with 'C' as centre of mass. Let 'I' be its moment of inertia about an axis PQ in its plane and I_{cm} the moment of inertia about a parallel axis RS passing through C. Let the distance between RS and PQ be 'r'.

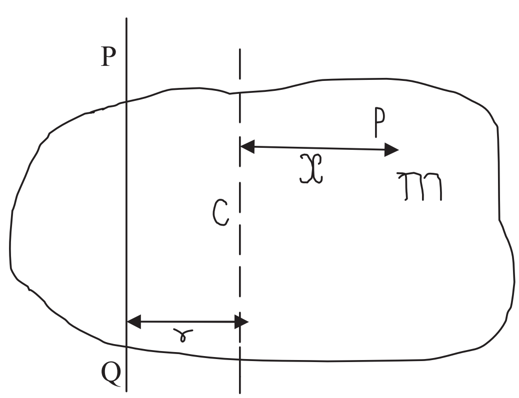


Figure 1

Let us consider a particle P of mass m at a distance x from RS. Its distance from PQ is (r + x) and its moment of inertia about it is $m(r + x)^2$. Therefore, the moment of inertia of the lamina about PQ is given by-

$I = \Sigma\, m(r + x)^2$

$= \Sigma\, m(r^2 + x^2 + 2rx)$

$= \Sigma\, mr^2 + \Sigma\, mx^2 + \Sigma\, 2mrx$

or $I = r^2\, \Sigma\, m + \Sigma\, mx^2 + 2r\, \Sigma\, mx$ …..(4)

(since r is constant)

But $\Sigma\, mx^2 = I_{cm}$, where I_{cm} is the moment of inertia of the lamina about RS, $r^2\, \Sigma\, m = r^2 M$ where M is the total mass of the lamina and $\Sigma\, mx = 0$ because the sum of the moments of all the mass particles of a body about an axis through the centre of mass of the body is zero. Hence, the equation (4) becomes

$I = r^2 M + I_{cm} + 0$

or $I = I_{cm} + M r^2$ …..(5)

It may be seen clearly from equation (5) that the moment of inertia of a body about an axis through the centre of mass is the least. The moment of inertia of the body about an axis not passing through the centre of mass is always greater than its moment of inertia about a parallel axis passing through the centre of mass of the body.

8.5.2 Theorem of Perpendicular Axes

According to this theorem, the moment of inertia of a uniform plane lamina about an axis perpendicular to its plane is equal to the sum of its moments of inertia about any two mutually perpendicular axes in its plane intersecting on the first axis.

If I_x and I_y be the moments of inertia of a plane lamina about two mutually perpendicular axes OX and OY in the plane of the lamina and I_z be its moment of inertia about an axis OZ, passing through the point of intersection O and perpendicular to the plane of the lamina, then

$I_z = I_x + I_y$

This is the "theorem of perpendicular axes".

Proof: Let OZ be the axis perpendicular to the plane of the lamina about which the moment of inertia is to be taken. Let OX and OY be two mutually perpendicular axes in the plane of the lamina and intersecting on OZ.

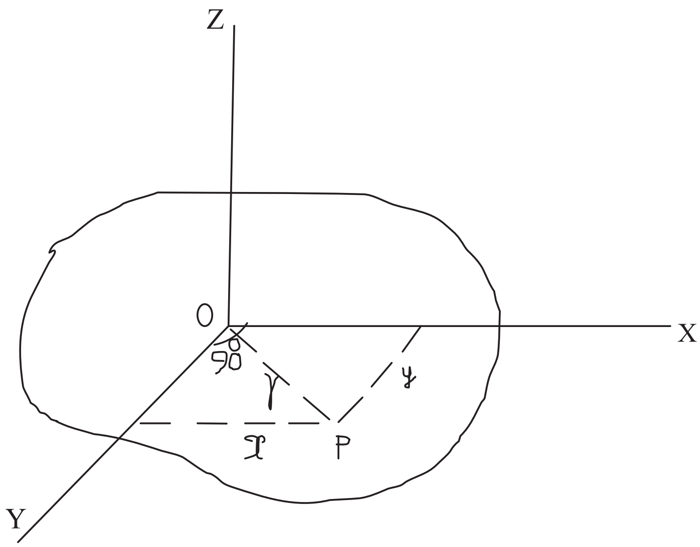


Figure 2

Let us consider a particle P of mass 'm' at a distance of 'r' from OZ. The moment of inertia of this particle about OZ is mr^2. Therefore, the moment of inertia I_z of the whole lamina about OZ is $I_z = \Sigma\, mr^2$

But $r^2 = x^2 + y^2$, where x and y are the distances of P from OY and OX respectively.

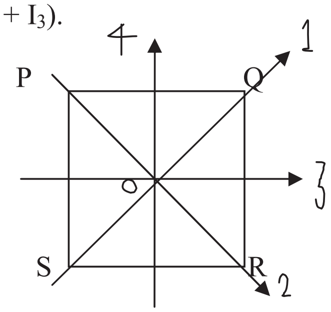
Therefore, $I_z = \Sigma\, m(x^2 + y^2) = \Sigma\, mx^2 + \Sigma\, my^2$

But $\Sigma\, mx^2$ is the moment of inertia I_y of the lamina about OY and $\Sigma\, my^2$ is the moment of inertia I_x of the lamina about OX.

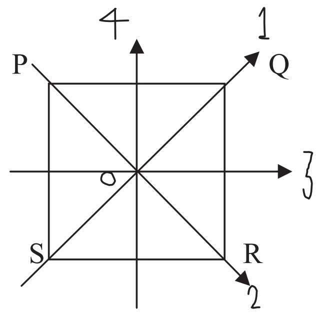
Therefore, $I_z = I_y + I_x$

or $I_z = I_x + I_y$

Example 3: Show that the moment of inertia I of a thin square plate PQRS (Figure) of uniform thickness about an axis passing through the centre O and perpendicular to the plane of the plate is $(I_1 + I_2)$ or $(I_3 + I_4)$ or $(I_1 + I_3)$.



Solution:



Let I_1, I_2, I_3 and I_4 are the moments of inertia about axes 1, 2, 3 and 4 respectively which are in the plane of the plate.

By the theorem of perpendicular axes, we have-

$I = I_1 + I_2 = I_3 + I_4$

By symmetry of square plate, $I_1 = I_2$ and $I_3 = I_4$

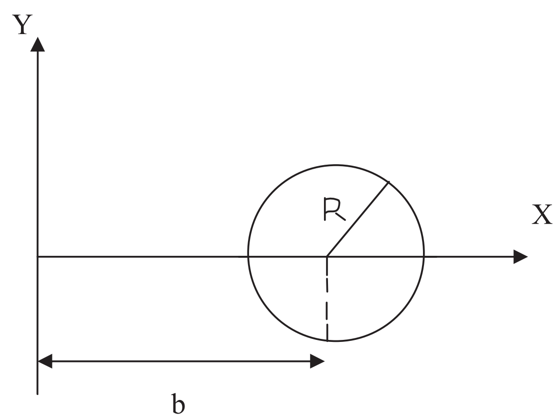
Therefore, $I = 2I_1 = 2\ I_3$

or $I_1 = I_3$

Thus $I_1 = I_2 = I_3 = I_4$

and $I = I_1 + I_2 = I_1 + I_3$

Self Assessment Question (SAQ) 4: The figure represents a disc of mass M and radius R, lying in XY- plane with its centre on X-axis at a distance 'b' from the origin. Determine the moment of inertia of the disc about Y-axis if its moment of inertia about a diameter is $\frac{MR^2}{4}$.



8.6 SUMMARY

In the present unit, we have studied about equations of motion and derived all the three equations of motion. In the unit, we have also studied Newton's laws of rotational motion. According to the first law of rotational motion "unless an external torque is applied to it, the state of rest or uniform rotational motion of a body about its fixed axis of rotation remains unaltered" while the second law states "the rate of change of angular momentum (or the rate of change of rotation) of a body about a fixed axis of rotation is directly proportional to the torque applied and takes place in the direction of the torque". According to Newton's third law of rotational motion "when a torque is applied by one body on another, an equal and opposite torque is applied by the latter on the former about the same axis of rotation". Sometimes it is difficult to calculate the moments of inertia of some specific bodies. In this unit, we have also studied and derived the general theorems on moment of inertia. These theorems are known as theorem of parallel axes and theorem of perpendicular axes. If 'I_{cm}' be the moment of inertia of a body about a parallel axis through its centre of mass, 'M' be the mass of the body and 'r' be the perpendicular distance between two axes, then moment of inertia of the body $I = I_{cm} + Mr^2$. This is the theorem of parallel axes. If I_x and I_y be the moments of inertia of a plane lamina about two mutually perpendicular axes OX and OY in the plane of the lamina and I_z be its moment of inertia about an axis OZ, passing through the point of intersection O and perpendicular to the plane of the lamina, then $I_z = I_x + I_y$. This is the theorem of perpendicular axes. These theorems make easy to find out the moments of inertia of those specific bodies. We have included examples and self assessment questions (SAQs) to check your progress.

8.7 GLOSSARY

Velocity- a vector physical quantity whose magnitude gives speed

Acceleration- increase of velocity

Unless- except, if not

External- exterior, outer

Rotational- the action of moving in a circle

Unaltered- unchanged, unaffected

Facilitate- make easy, smooth the progress of

8.8 TERMINAL QUESTIONS

1. Explain the equations of motion.

2. What are Newton's laws of rotational motion? Explain.

3. Discuss and derive general theorems on moment of inertia.

4. Calculate the moment of inertia of mass M and length L about an axis perpendicular to the length of the rod and passing through a point equidistant from its midpoint and one end.

5. Give the statement of theorem of parallel axes. Also derive this theorem.

6. State and establish the theorem of perpendicular axes.

8.9 ANSWERS

Self Assessment Questions (SAQs):

1. Since the car is moving with constant speed, therefore acceleration of car a = 0.

Here, u = 30 Km/hr = 25/3 m/sec, t = 1 hr = 3600 sec

Using second equation of motion, $s = ut + \frac{1}{2} a t^2$

$s = (25/3) \times 3600 + \frac{1}{2}(0) \times (3600)^2 = 3 \times 10^4$ m = 30 Km

2. Given $u = 6 \times 10^6$ m/sec, $a = - 1.26 \times 10^{14}$ m/sec^2, v = 0

Using third equation of motion, $v^2 = u^2 + 2as$

$(0)^2 = (6 \times 10^6)^2 + 2(- 1.26 \times 10^{14})\, s$

or s = 0.143 m

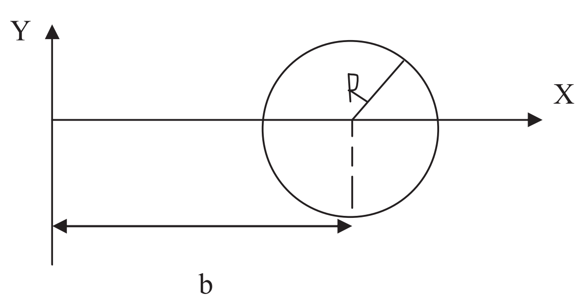
3. Here acceleration = at^2

Using v = u + at

$v = u + (at^2)\, t = u + at^3$

Hence relation (c) is valid.

4.



By the theorem of parallel axes, the moment of inertia of disc about Y-axis is-

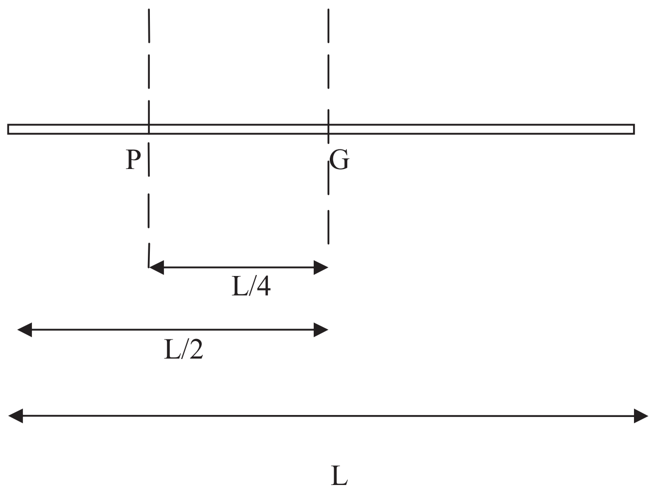
$I = I_{cm} + Mb^2$

$= \frac{MR^2}{4} + Mb^2$

$= M\left(\frac{R^2}{4} + b^2\right)$

Terminal Questions:

4. The moment of inertia of rod about an axis passing through centre of mass and perpendicular to length $I_{cm} = \frac{ML^2}{12}$



The moment of inertia of rod about an axis passing through P and perpendicular to length by theorem of parallel axes is-

$I = I_{cm} + Mr^2$

Here r = PG = L/4

Therefore, $I = (ML^2/12) + M(L/4)^2 = (7/48)\,ML^2$

8.10 REFERENCES

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2. Mechanics- JK Ghose, Shiva Lal Agarwal and Company, Delhi

3. Elements of Mechanics- JP Agrawal and Satya Prakash, Pragati Prakashan, Meerut

8.11 SUGGESTED READINGS

1. Concepts of Physics, Part I, HC Verma, Bharati Bhawan, Patna

2. Mechanics and Wave Motion – DN Tripathi and RB Singh, Kedar Nath Ram Nath, Meerut

3. Modern Physics, Beiser, Tata McGraw Hill

4. Fundamentals of Physics, David Halliday, Robert Resnick, Jearl Walker, John Wiley & Sons

UNIT 9: FORMULATION OF MOMENT OF INERTIA

Structure

9.1 Introduction

9.2 Objectives

9.3 Formulation and Derivation of Moment of Inertia

9.3.1 Moment of Inertia of a Thin Uniform Rod

9.3.2 Moment of Inertia of a Rectangular Lamina

9.3.3 Moment of Inertia of a Circular Lamina

9.3.4 Moment of Inertia of a Solid Sphere

9.3.5 Moment of Inertia of a Solid Cylinder

9.4 Summary

9.5 Glossary

9.6 Terminal Questions

9.7 Answers

9.8 References

9.9 Suggested Readings

9.1 INTRODUCTION

In the previous unit, we have studied theorem of parallel axes and theorem of perpendicular axes. These theorems make easy the calculations of moment of inertia in some typical cases. In general, the moment of inertia of a body is calculated as the sum $\Sigma\ mr^2$ for all particles making up the body by integration or other means. In this unit, we shall formulate and derive the moment of inertia for some simple symmetric systems like rod, rectangular lamina, circular lamina, solid sphere and cylinder.

9.2 OBJECTIVES

After studying this unit, you should be able to-

- Understand the formulation and derivation of moment of inertia
- Solve problems based on moment of inertia
- Apply the formulae of moment of inertia

9.3 FORMULATION AND DERIVATION OF MOMENT OF INERTIA

The moment of inertia of a continuous homogeneous body of a definite geometrical shape can be calculated by (i) first obtaining an expression for the moment of inertia of an infinitesimal element of the same shape of mass dm about the given axis i.e. $dm.r^2$, where r is the distance of the infinitesimal element from the axis and then (ii) integrating this expression over appropriate limits so as to cover the whole body. In fact sometimes the theorem of parallel and perpendicular axes are also used to calculate the moment of inertia about an axis when the moment of inertia of the body about some other axis has first been calculated. Thus,

$$I = \int dm.r^2$$

where the integral is taken over the whole body.

9.3.1 Moment of Inertia of a Thin Uniform Rod

(i) About an axis passing through its centre of mass and perpendicular to its length

Let PQ be a thin uniform rod of mass per unit length m. Let RS be the axis passing through the centre of mass C of the rod and perpendicular to its length PQ.

Let us consider an element of length dr at a distance r from centre of mass C.

The mass of the element, $dm = m.dr$

The moment of inertia of element about the axis through C $= dm.r^2$

$= m\,dr\,r^2$

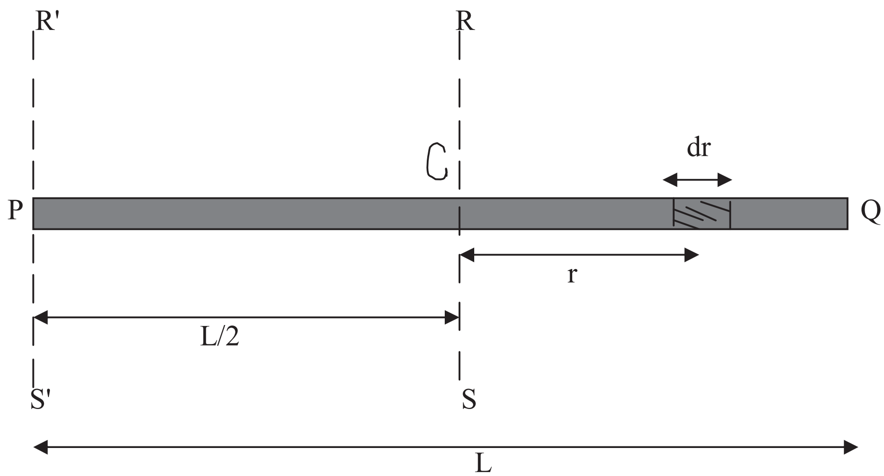

Figure 1

The moment of inertia of the whole rod about axis RS is the sum of the moments of inertia of all such elements lying between r = - L/2 at P and r = L/2 at Q. Hence the moment of inertia

$$I_{cm} = \int_{-L/2}^{L/2} mr^2 dr$$

$$= 2m \int_0^{L/2} r^2 dr = \frac{mL^3}{12} = \frac{(mL)L^2}{12}$$

$$= \frac{ML^2}{12}$$

where mL = M, the total mass of the rod

Thus, $I_{cm} = \frac{ML^2}{12}$ …..(1)

(ii) About an axis passing through its one end and perpendicular to its length

Let $R'\,S'$ be the axis passing through the end P of the rod (Figure 1). The moment of inertia I about a parallel $R'\,S'$ axis passing through one end (using theorem of parallel axes)-

$$I = I_{cm} + M\,(CP)^2$$

$$= \frac{ML^2}{12} + M\left(\frac{L}{2}\right)^2$$

$$= \frac{ML^2}{3}$$

9.3.2 Moment of Inertia of a Rectangular Lamina

(i) About an axis in its own plane, parallel to one of the sides and passing through the centre of mass

Let PQRS be a rectangular lamina of mass M, length l and breadth b with O as its centre of mass. Let the mass per unit area of the lamina is σ. Let us consider a strip of width dr parallel to the given axis Y' at a distance r from it.

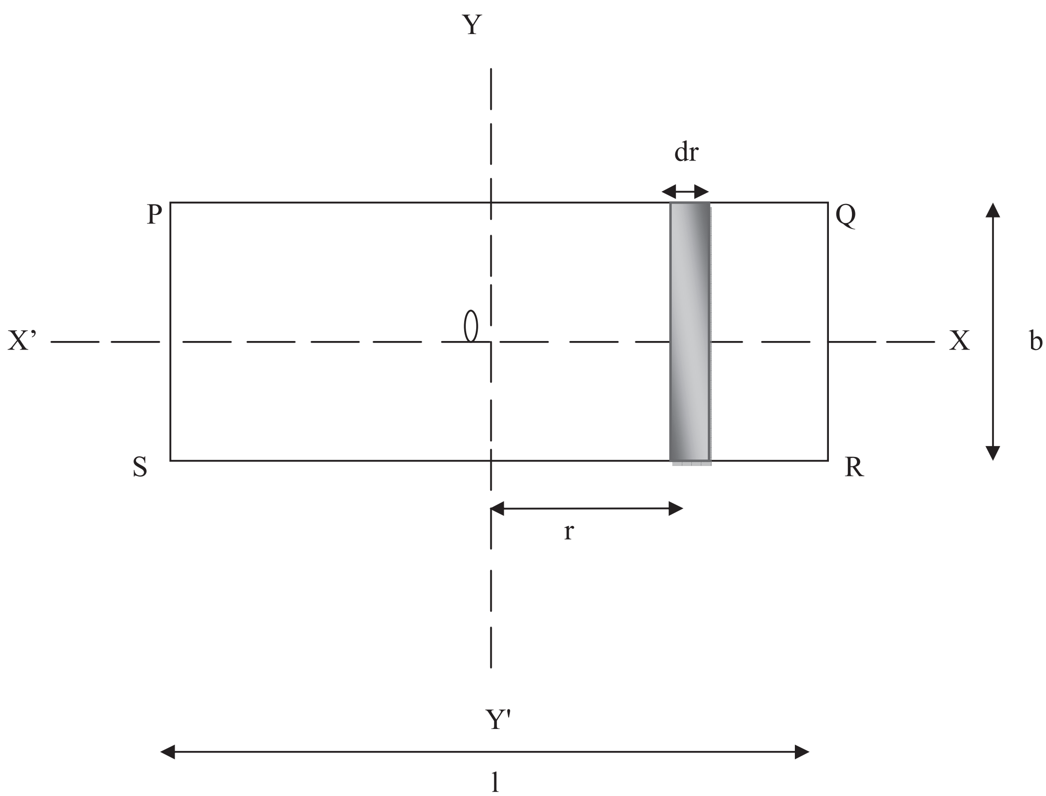

Figure 2

Area of the strip = b dr

Mass of the strip m = (bdr) σ

Moment of inertia of strip about the axis YY' = mr^2

$= (bdr)\,\sigma\, r^2$

$= \sigma\, b\, r^2\, dr$

The moment of inertia of the whole lamina about axis YY′

$I_y = \int_{-l/2}^{l/2} \sigma b r^2 dr = 2\int_0^{l/2} \sigma b r^2 dr$

$= 2\sigma b\int_0^{l/2} r^2 dr = \frac{\sigma bl^3}{12} = \frac{(\sigma bl)l^2}{12}$

i.e. $I_y = \frac{Ml^2}{12}$ …..(1)

where σbl = M, the total mass of the lamina

Similarly, the moment of inertia of the lamina about an axis XX' parallel to the side of length l and passing through the centre will be-

$I_x = \frac{Mb^2}{12}$ …..(2)

(ii) About an axis perpendicular to its plane and passing through the centre of mass

The moment of inertia of the lamina about an axis passing through the centre C and perpendicular to the plane of the lamina

$I = I_x + I_y$ (using theorem of perpendicular axes)

$= \frac{Mb^2}{12} + \frac{Ml^2}{12}$

$= M\left(\frac{l^2+b^2}{12}\right)$ …..(3)

9.3.3 Moment of Inertia of a Circular Lamina

(i) About an axis passing through its centre and perpendicular to its plane

Let O be the centre of the circular lamina and σ the mass per unit area. Let the lamina be supposed to be composed of a number of thin circular strips. Let us consider one such strip of radius r and thickness dr.

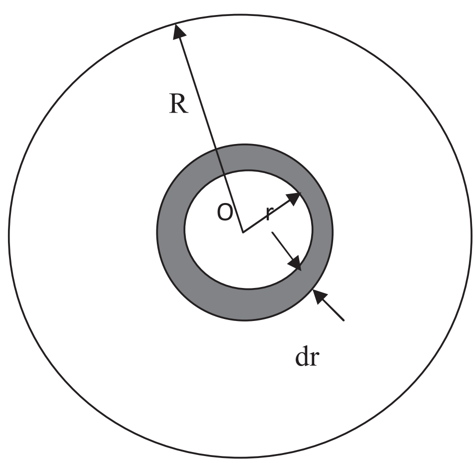

Figure 3

The circumference of the strip = 2π r

Area of the strip = 2π r dr

Mass of the strip m = σ (2π r dr)

The moment of inertia of the strip about an axis passing through O and perpendicular to the plane of the lamina dI = mass × (distance)^2 = m × r^2

$$= \sigma(2\pi r\, dr)\, r^2$$

$$= \sigma\, (2\pi r^3\, dr)$$

The moment of inertia of whole lamina I = ∫dI

$$= \int \sigma\, (2\pi r^3\, dr)$$

or $I = 2\pi\sigma \int r^3\, dr$ …..(1)

If circular lamina is solid, then its moment of inertia $I = 2\pi\sigma\int_0^R r^3 dr$

$$= 2\pi\sigma\,\frac{R^4}{4} = \frac{1}{2}R^2(\pi R^2\sigma)$$

$$= \frac{1}{2}R^2M$$

where $\pi R^2\sigma = M$, mass of the disc

Thus, $I = \frac{1}{2} M R^2$ …..(2)

If lamina is having concentric hole and R' and R be the its internal and external radii then the moment of inertia $I = 2\pi\sigma\int_{R'}^{R} r^3 dr$

$$= \frac{2\pi\sigma}{4}[R^4 - R'^4]$$

$$= \frac{2\pi\sigma}{4}(R^2 + R'^2)\,(R^2 - R'^2)$$

$$= \pi(R^2 - R'^2)\,\sigma\left\{\frac{1}{2}(R^2 + R'^2)\right\}$$

$$= \frac{1}{2}M\,(R^2 + R'^2)$$

where $\pi(R^2 - R'^2)\,\sigma = M$, mass of the lamina

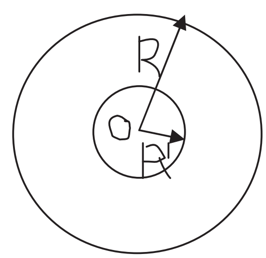

Figure 4

Thus, $I = \frac{1}{2} M\,(R^2 + R'^2)$ …..(3)

(ii) About any diameter

Let us consider two mutually perpendicular diameters AB and CD of the lamina. The lamina is symmetrical about both diameters AB and CD.

In accordance with the theorem of perpendicular axis, the moment of inertia about diameter $I_D = \frac{I}{2}$, where I is the moment of inertia of the disc, about an axis. Through it's center and perpendicular to it's plane.

For solid lamina, $I_D = \frac{1}{2}\left(\frac{1}{2}MR^2\right)$

$$= \frac{1}{4}MR^2$$

For circular lamina with concentric hole,

$I_D = \frac{1}{4}M(R^2 + R'^2)$

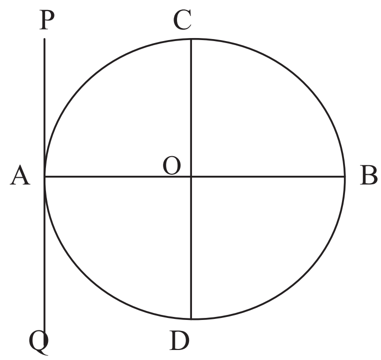

Figure 5

(iii) About a tangent in its plane

Let PQ be a tangent to the lamina in its plane and let it be parallel to the diameter CD.

Using theorem of parallel axes, the moment of inertia of lamina about PQ = Moment of inertia of lamina about CD + $(OA)^2$

or $I_T = I_D + MR^2$. M.

For a solid lamina, $I_T = \frac{1}{4}MR^2 + MR^2$

$= \frac{5}{4}MR^2$

For a lamina having a concentric hole,

$I_T = \frac{1}{4}M(R^2 + R'^2) + MR^2$

$= \frac{1}{4}M(5R^2 + R'^2)$

(iv) About a tangent perpendicular to its plane

Using theorem of parallel axes, moment of inertia of lamina about a tangent perpendicular to its plane $I'_T = I + MR^2$

For a solid lamina, $I'_T = \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$

For a circular lamina having concentric hole, $I'_T = \frac{1}{2}M(R^2 + R'^2) + MR^2$

$= = \frac{1}{2}M(3R^2 + R'^2)$

9.3.4 Moment of Inertia of a Solid Sphere

(i) About a diameter

Let us consider a sphere of radius R and mass M with centre at O. Let ρ be the density of the material of the sphere.

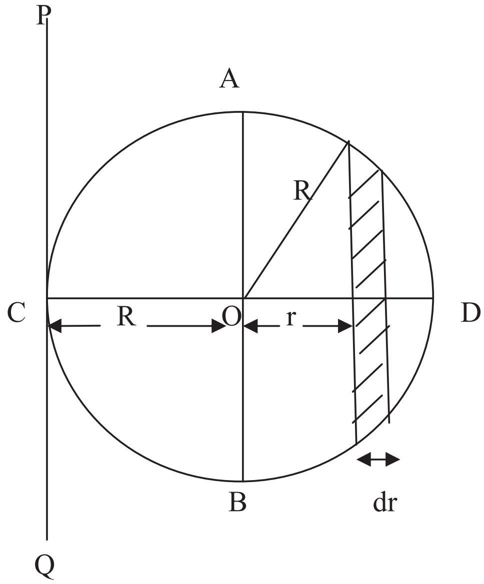

Figure 7

Let us divide the sphere into a number of thin discs by planes perpendicular to the diameter CD and let us consider one such elementary disc of thickness dr at a distance r from O.

Obviously, the radius of the elementary disc = $\sqrt{R^2 - r^2}$

Volume of the elementary disc = its area × its thickness

$= \pi(R^2 - r^2)\, dr$

Mass of the elementary disc m = volume × density

$= \pi(R^2 - r^2)\, dr\, \rho$

The moment of inertia of the disc about diameter CD-

$dI = \frac{1}{2}$ [mass of the disc × (radius of the disc)^2]

$= \frac{1}{2}\pi(R^2-r^2)\ dr\ \rho\times (R^2-r^2)$

$= \frac{1}{2}\pi(R^2-r^2)^2\ dr\ \rho$ …..(1)

The moment of inertia of the whole sphere about CD, $I = \int_{-R}^{+R} dI$

$= \int_{-R}^{+R} \frac{1}{2}\pi(R^2 - r^2)^2\rho\ dr$

$= \int_0^R \pi(R^2 - r^2)^2\rho\ dr$

$= \pi\rho\int_0^R (R^4 + r^4 - 2R^2r^2)dr$

$= \frac{8}{15}\pi\rho\ R^5$

$= \frac{2}{5}\left(\frac{4}{3}\pi R^3\rho\right)R^2$

$= \frac{2}{5}M\ R^2$, where $\frac{4}{3}\pi R^3\rho = M$, the mass of the sphere

Therefore, $I = \frac{2}{5}M\ R^2$

(ii) About a tangent

Let PQ be a tangent to the sphere parallel to the diameter AB and at a distance R (the radius of the sphere) from it.

Using theorem of parallel axes, the moment of inertia of the solid sphere about a tangent PQ-

I_T = moment of inertia of sphere about CD + $M\times(OC)^2$

$= \frac{2}{5}M\ R^2 + MR^2 = \frac{7}{5}MR^2$

9.3.5 Moment of Inertia of a Solid Cylinder

Let us consider a solid cylinder of mass M, radius R and length L.

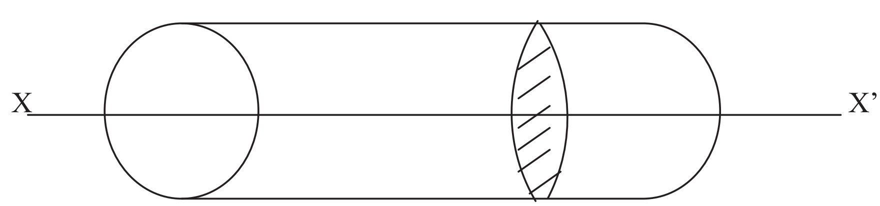

Figure 8